

Analysis of Radial Vibration Considering the Phase Shift of Modulated Electromagnetic Excitations in FSCW PMSMs for Robot Joint Applications

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Abstract—In this article, the phase characteristics of the lowest spatial order modulated excitations and their influence on radial vibration are analyzed for fractional-slot concentrated winding (FSCW) permanent magnet synchronous motors (PMSMs) with pole–slot combinations satisfying $2p = S \pm 2$, which are widely adopted in robot joint applications. The phase shifts of the modulated radial and tangential forces, and moment are analytically derived by considering tangential air-gap electromagnetic force density with respect to the slot opening pitch. Then, 2-D finite-element analysis is used to characterize the phase relationships of the lowest spatial order modulated excitations under no-load and load conditions. The results show that the excitation phases depend on the slot opening pitch under no-load conditions, whereas they are primarily influenced by armature reaction under load conditions. Accordingly, the phase-dependent radial vibration contributions of the modulated excitations are established for different slot opening pitches and pole–slot combinations. The proposed analysis framework is further applied to FSCW PMSMs with different pole–slot combinations with $2p = S \pm 2$ to assess the generality of the observed phase characteristics. Finally, radial vibration contributions under no-load and load conditions are verified through vibration experiments.

Index Terms—Force modulation effect, phase shift, pole–slot combination, radial vibration, robotic joints, slot opening pitch.

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I. INTRODUCTION

HUMANOID robots require joint actuators that provide high torque density, smooth motion, and low vibration over a wide range of operating conditions [1], [2]. In such applications, radial vibration in electric machines can directly degrade positioning accuracy, increase acoustic noise, and deteriorate interaction quality between the robot and its environment [3]. As a result, vibration characteristics have become a critical design consideration for joint motors. Fractional-slot concentrated winding (FSCW) permanent magnet synchronous motors (PMSMs) have been widely adopted in robotic joint applications due to their compact structure, high torque density, and manufacturability [4]. The characteristics of electromagnetic (EM) excitation in PMSMs are strongly influenced by the pole–slot combination. The lowest spatial order of the air-gap EM force density (AEFD) is determined by the greatest common divisor of pole and slot numbers [5], and the radial vibration is inversely proportional to the fourth power of this spatial order [6]. Consequently, pole–slot combinations satisfying $2p = S \pm 2$, which provide a high fundamental winding factor while avoiding the lowest spatial order of AEFDs equal to 1, are widely used in robotic joint applications [7], [8], [9]. Here, p and S are the number of pole pairs and stator slots, respectively. Therefore, this article focuses on FSCW PMSMs with $2p = S \pm 2$ pole–slot combinations, and the proposed analysis is also applicable to similar configurations beyond robotic joint applications.

Recently, the force modulation effect, which transforms high spatial order AEFD components into low spatial orders, has been actively investigated in FSCW PMSMs. Among these studies, the contribution of vibration was analyzed by considering the phase of modulated EM excitations [10], [11], [12]. However, these studies were limited to a single pole–slot combination, with the modulated moment neglected in [10], and only the modulated radial force considered in [11] and [12]. Other studies analyzed the vibration characteristics of FSCW PMSMs with different pole–slot combinations considering the force modulation effect [13], [14], [15], [16], [17]. It was established in [13] that the $2p$ th spatial order AEFD, which has the largest amplitude, is more significantly modulated into the $(2p - S)$ th spatial order in the 12-slot 10-pole (12S10P) than in the 12S14P

at twice the fundamental electrical frequency (i.e., $2f_e$). However, a comprehensive analysis of vibration contributions based on the phase relationships of the modulated EM excitations remains insufficient. In [14] and [15], the slot opening pitch was identified as a critical factor affecting the force modulation, and its influence on vibration characteristics was systematically investigated. However, Zhu et al. [14] considered only the amplitude of the modulated radial force, and the phase shift of modulated excitations under load conditions was not addressed in [15]. The phase shift of the modulated radial and tangential forces under load conditions was analyzed in [16]. However, the modulated moment was not considered. Although the modulated moment was considered in [17], the phase relationships among the modulated excitations and their contributions to vibration were not clearly clarified. Various vibration reduction design methods for FSCW PMSMs have also been proposed, such as unequal tooth and permanent magnet (PM) flux density harmonic injection [18], PM configurations that symmetrize the magnetic field [19], and piecewise staggered unequal poles [20]. However, the influence of pole–slot combinations on the force modulation effect was not analyzed, with [18] and [19] considering only the modulated radial force and [20] neglecting the modulated moment.

Therefore, it is essential to analyze the vibration characteristics of the FSCW PMSMs with $2p = S \pm 2$ used in robotic joint motors by explicitly considering the phase relationships among the modulated radial and tangential forces, and moment. The phase relationships of these modulated EM excitations vary not only under load conditions but also with the slot opening pitch, leading to different vibration contributions from each excitation. In this article, the phase shifts of the modulated radial and tangential forces, and moment in FSCW PMSMs with $2p = S \pm 2$ are analyzed according to load conditions and slot opening pitches. Based on this analysis, the vibration contributions of each excitation component are clarified for different pole–slot combinations, and configurations favorable for low-vibration robotic joint applications are identified. The main contributions of this article are summarized as follows.

- 1) The phase characteristics of the modulated radial and tangential forces, and moment in FSCW PMSMs are analytically derived by considering the tangential AEFd with respect to slot opening and pole–slot combinations.
- 2) The phase-dependent vibration contributions are clarified by establishing the relationship between phase shifts of the modulated excitations and resulting radial vibration under both no-load and load conditions.
- 3) The vibration characteristics of FSCW PMSMs with $2p = S \pm 2$ are systematically analyzed and experimentally validated, providing guidelines for pole–slot combination selection in low-vibration robot joint motors.

The rest of this article is organized as follows. Section II presents the analytical formulation of the modulated radial and tangential forces, and moment in FSCW PMSMs by considering the tangential AEFd, and investigates their phase characteristics under no-load and load conditions using finite-element analysis (FEA). Section III analyzes the influence of the phase shifts of the modulated excitations on radial vibration, and discusses the

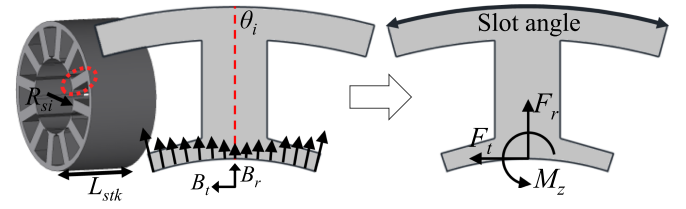


Fig. 1. Equivalence of distributed AEFds to modulated forces and moment.

resulting vibration contributions for different pole–slot combinations with $2p = S \pm 2$. Section IV experimentally validates the predicted radial vibration characteristics under both no-load and rated load conditions. Finally, Section V concludes the article.

II. ANALYSIS OF MODULATED EM EXCITATIONS

In FSCW PMSMs, the dominant pole-frequency vibration is primarily induced by the lowest spatial order radial and tangential AEFds and the force modulation of $2p$ th spatial order AEFd [16], [20]. Therefore, this section focuses on the analysis of the lowest spatial order modulated radial and tangential forces, and moment at $2f_e$, which have the most significant influence on pole-frequency vibration.

A. Modulated Radial and Tangential Forces, and Moment Derived From Radial and Tangential AEFds

In FSCW PMSMs, the EM excitations acting on the stator originate from the AEFds, which can be derived from the Maxwell stress tensor. The radial and tangential AEFds can be expressed as

$$\sigma_r(\theta, t) = \frac{B_r^2(\theta, t) - B_t^2(\theta, t)}{2\mu_0} \quad (1)$$

$$\sigma_t(\theta, t) = \frac{B_r(\theta, t)B_t(\theta, t)}{\mu_0} \quad (2)$$

where σ_r and σ_t are the radial and tangential AEFds, respectively, B_r and B_t are the radial and tangential air-gap magnetic flux densities, respectively, θ is the circumferential position, t is the time, and μ_0 is the vacuum permeability. For the analysis of modulated excitations, the distributed AEFds acting on each stator tooth are commonly transformed into equivalent concentrated excitations through circumferential integration over the effective tooth pitch. As illustrated in Fig. 1, the distributed AEFds acting on a stator tooth are equivalently represented by a modulated radial and tangential forces, and moment applied at the tooth center. Accordingly, the equivalent modulated radial and tangential forces, and moment acting on the i th stator tooth can be written as [21]

$$F_r(i, t) = R_{si} L_{stk} \int_{\theta_i - \pi\beta/S}^{\theta_i + \pi\beta/S} \{\sigma_r \cos(\theta_i - \theta) + \sigma_t \sin(\theta_i - \theta)\} d\theta \quad (3)$$

$$F_t(i, t) = R_{si} L_{stk} \int_{\theta_i - \pi\beta/S}^{\theta_i + \pi\beta/S} \{\sigma_t \cos(\theta_i - \theta) - \sigma_r \sin(\theta_i - \theta)\} d\theta \quad (4)$$

$$M_z(i, t) = R_{si}^2 L_{stk} \int_{\theta_i - \pi\beta/S}^{\theta_i + \pi\beta/S} \{\sigma_r \sin(\theta_i - \theta) - \sigma_t \cos(\theta_i - \theta) + \sigma_t\} d\theta \quad (5)$$

where F_r , F_t , and M_z are the modulated radial and tangential forces, and moment, respectively, R_{si} is the stator inner radius, L_{stk} is the stack length, θ_i is the angular position of the center of the i th tooth, and β is the effective tooth pitch, which is defined as $\beta = 1 - \rho_{so}$, where ρ_{so} is the slot opening pitch. The slot opening pitch is defined as the ratio of the slot opening angle to the slot angle ($2\pi/S$). The radial and tangential AEFDs can be decomposed into their spatial harmonic components of spatial order n at $2f_e$. To identify the relative phase relationship of the modulated excitations by considering the tangential AEFD, the modulated excitations produced by the n th spatial order harmonic components of the radial and tangential AEFDs can be derived as

$$\begin{aligned} F_r(i) &= R_{si} L_{stk} \int_{\theta_i - \pi\beta/S}^{\theta_i + \pi\beta/S} \{\sigma_{r,n} \cos(n\theta) \cos(\theta_i - \theta) + \sigma_{t,n} \sin(n\theta) \sin(\theta_i - \theta)\} d\theta \\ &= 2R_{si} L_{stk} \cos(n\theta_i) \{\sigma_{r,n} K_1(\rho_{so}) + \sigma_{t,n} K_2(\rho_{so})\} \end{aligned} \quad (6)$$

$$\begin{aligned} F_t(i) &= R_{si} L_{stk} \int_{\theta_i - \pi\beta/S}^{\theta_i + \pi\beta/S} \{\sigma_{t,n} \sin(n\theta) \cos(\theta_i - \theta) - \sigma_{r,n} \cos(n\theta) \sin(\theta_i - \theta)\} d\theta \\ &= 2R_{si} L_{stk} \sin(n\theta_i) \{\sigma_{t,n} K_1(\rho_{so}) - \sigma_{r,n} K_2(\rho_{so})\} \end{aligned} \quad (7)$$

$$\begin{aligned} M_z(i) &\approx R_{si}^2 L_{stk} \int_{\theta_i - \pi\beta/S}^{\theta_i + \pi\beta/S} \{\sigma_{r,n} \cos(n\theta) \sin(\theta_i - \theta)\} d\theta \\ &= 2R_{si}^2 L_{stk} \sin(n\theta_i) \sigma_{r,n} K_2(\rho_{so}) \end{aligned} \quad (8)$$

where $\sigma_{r,n}$ and $\sigma_{t,n}$ are the amplitudes of the n th spatial harmonic components of the radial and tangential AEFDs at $2f_e$, respectively. In (8), the terms associated with the tangential AEFD almost cancel each other out and are therefore neglected in the derivation [17]. The modulation coefficients are defined as

$$K_1(\rho_{so}) = \frac{1}{2} \left\{ \frac{\sin((n-1)\alpha)}{n-1} + \frac{\sin((n+1)\alpha)}{n+1} \right\} \quad (9)$$

$$K_2(\rho_{so}) = \frac{1}{2} \left\{ \frac{\sin((n-1)\alpha)}{n-1} - \frac{\sin((n+1)\alpha)}{n+1} \right\} \quad (10)$$

where

$$\alpha = \frac{\pi}{S} (1 - \rho_{so}). \quad (11)$$

Equations (6)–(11) show that the slot opening pitch directly modifies the amplitudes and phase characteristics of the modulated radial and tangential forces, and moment through the modulation coefficients. The spatial distributions of the modulated excitations are governed by either $\cos(n\theta_i)$ or $\sin(n\theta_i)$. The modulation coefficients K_1 and K_2 represent the influence of the slot opening pitch on the modulation of the n th spatial order AEFD. These coefficients jointly determine the amplitude

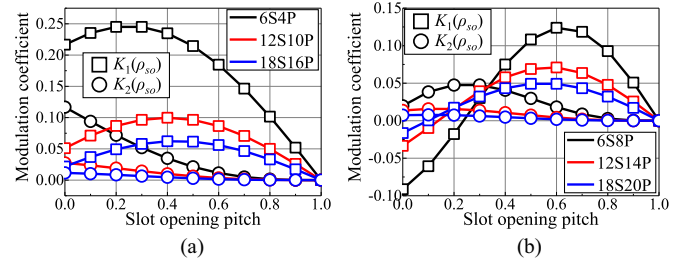


Fig. 2. Modulation coefficients of the $2p$ th spatial order AEFD for the $2p = S \pm 2$ combinations. (a) $2p = S - 2$. (b) $2p = S + 2$.

TABLE I
PHASE OF THE LOWEST SPATIAL ORDER MODULATED EXCITATIONS OF
 $2P = S \pm 2$ UNDER NO-LOAD CONDITION AT $2f_e$

Items	$2p = S - 2$		$2p = S + 2$	
	$\rho_{so} < \rho_{min}$	$\rho_{so} > \rho_{min}$	$\rho_{so} < \rho_{min}$	$\rho_{so} > \rho_{min}$
Modulated radial force	π	π	0	π
Modulated tangential force	$\pi/2$	$3\pi/2$	$3\pi/2$	$\pi/2$
Modulated moment	$3\pi/2$	$3\pi/2$	$\pi/2$	$\pi/2$

and sign of the modulated force components, thereby regulating the amplitude and phase characteristics of the resulting modulated excitations. According to the Nyquist–Shannon sampling theorem, aliasing occurs when the spatial order n of an AEFD component exceeds half of the number of stator slots, $|n| > S/2$ [11]. Then, the corresponding excitation is modulated into a lower spatial order component of $(|n| - kS)$, where k is an integer. Considering the $2p$ th spatial order AEFD has the largest amplitude [15], the phase relationships of the lowest spatial order modulated radial and tangential forces, and moment are established based on the $2p$ th spatial order AEFD. Fig. 2 illustrates the modulation coefficients of the $2p$ th spatial order AEFD for pole–slot combinations with $2p = S \pm 2$. It can be observed that the modulation coefficient $K_1(\rho_{so})$ is larger than $K_2(\rho_{so})$ over most of the slot opening pitch range. However, since the amplitude of the radial AEFD is typically larger than that of the tangential AEFD, the sign of the modulated tangential force cannot be determined solely from the modulation coefficients. Therefore, considering that the $2p$ th spatial order radial AEFD is relatively larger than the corresponding tangential AEFD [10], [16], the phase relationships of the modulated excitations are determined accordingly. When the sign of a modulated excitation changes, the corresponding excitation undergoes a phase shift of π . In particular, the modulated force exhibits a π -phase shift around the critical slot opening pitch ρ_{min} , where the amplitude becomes zero. Based on the above procedure, the relative phase relationships of the lowest spatial order modulated excitations under no-load conditions at $2f_e$ are derived and summarized in Table I for $2p = S \pm 2$ pole–slot combinations.

B. Phase Characteristics of Modulated EM Excitations

Using FEA, the phase relationships of the lowest spatial order modulated excitations derived in Section II-A are verified under no-load conditions and further analyzed under load conditions. The configurations of the analyzed FSCW PMSMs are given

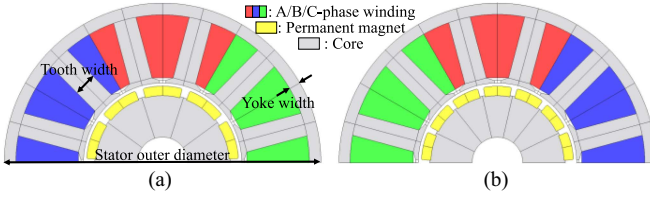


Fig. 3. Configurations for $2p = S \pm 2$ FSCW PMSMs. (a) 12S10P. (b) 12S14P.

TABLE II
SPECIFICATIONS OF THE 12S10P AND 12S14P PMSMs

Items	Values	Units
Stator outer diameter	100.0	mm
Yoke width	3.65	mm
Tooth width	7.3	mm
Slot opening pitch	0.1 - 0.45	-
Number of series turns per phase	116	-
Air-gap length	1.0	mm
Rotor outer diameter	48.0	mm
Pole-arc coefficient	0.833	-
PM thickness	3.0	mm
Stack length	50.0	mm
Slot fill factor	40.0	%
PM remanence	1.32	T(@50°C)
Rated torque	5.0	Nm
Rated rotational speed	1,000	rpm

in Fig. 3, and their specifications are given in Table II. For a fair comparison, the two PMSMs are designed with the same stator geometry and pole-arc coefficient. First, the air-gap radial and tangential magnetic flux densities are obtained from 2-D EM FEA by extracting the flux density components along a circular path at the midline of the stator-side air-gap. The radial and tangential AEFDs are calculated in the spatial and time domains using (1) and (2). The time-varying modulated radial and tangential forces, and moment acting on each stator tooth are then evaluated using (3)–(5). To identify the lowest spatial order modulated excitations that dominantly contribute to pole-frequency vibration, a 2-D fast Fourier transform (FFT) is applied to the modulated radial and tangential forces, and moment with respect to the tooth spatial position and time. Fig. 4 presents the amplitude and phase of the lowest spatial order modulated excitations for the 12S10P and 12S14P PMSMs under no-load condition at $2f_e$, according to the slot opening pitch. Since the FEA results include harmonic components other than the $2p$ th spatial order AEFD, the absolute phases differ from those in Table I, while the relative phase relationships remain consistent with the analytical results. Moreover, a π -phase shift is observed at $\rho_{\min}$, where the amplitude of the corresponding modulated excitation becomes zero. In [16], the phase of the lowest spatial order EM excitation was shown to vary under load conditions due to the armature reaction. Based on this observation, this article analyzes the load-induced phase shifts of the lowest spatial order modulated excitations relative to the no-load condition. When a load is applied, the armature reaction introduces an additional q -axis component that is orthogonal to the no-load excitation. Consequently, the modulated excitations

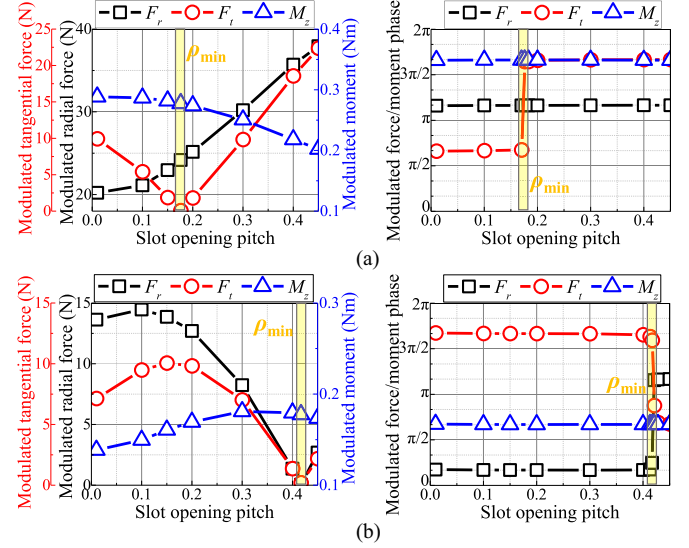


Fig. 4. Amplitude and phase of the lowest spatial order modulated excitations for $2p = S \pm 2$ FSCW PMSMs under no-load condition, according to the slot opening pitch. (a) 12S10P. (b) 12S14P.

under load conditions can be expressed as the vector sum of the no-load excitation and armature reaction component. Because the load-induced component is approximately orthogonal to the no-load excitation, the resulting excitation vector is rotated by nearly $\pm\pi/2$ relative to the no-load condition. The direction of this rotation, i.e., whether the phase leads or lags, is determined by the signs of the corresponding amplitude terms in (6)–(8). Furthermore, when the sign of the no-load modulated excitation reverses across $\rho_{\min}$, the direction of the phasor rotation changes accordingly. To verify this behavior, 2-D FEA was performed to evaluate the amplitude and phase of the lowest spatial order modulated excitations under load conditions. In humanoid robots, joint motors are required to generate high torque levels determined by rigid-body dynamics under highly dynamic motions rather than steady-state rated operation [2], [7], [8]. Accordingly, the load torque was investigated up to 20 Nm, exceeding the rated torque. Magnetic saturation under high load conditions may affect the absolute amplitudes of EM force harmonics, while the harmonic orders and their relative amplitudes remain similar to those in the linear case [22]. Accordingly, the phase relationships of the modulated excitations can still be analyzed under saturation. Fig. 5 shows the amplitude and phase of the lowest spatial order modulated excitations for the 12S10P and 12S14P PMSMs under different load torques in the slot opening pitch regions $\rho_{so} < \rho_{\min}$ and $\rho_{so} > \rho_{\min}$. As the slot opening pitch crosses $\rho_{\min}$, the rotation direction reverses for the modulated tangential force in the 12S10P PMSM and for both the modulated radial and tangential forces in the 12S14P PMSM, whereas the other excitations maintain their rotation direction as the load torque increases. The corresponding phase relationships under load conditions at $2f_e$ are given in Table III. In terms of amplitude, the lowest spatial order modulated radial and tangential forces increase with load torque for both the 12S10P and 12S14P PMSMs, while the modulated moment slightly decreases in the

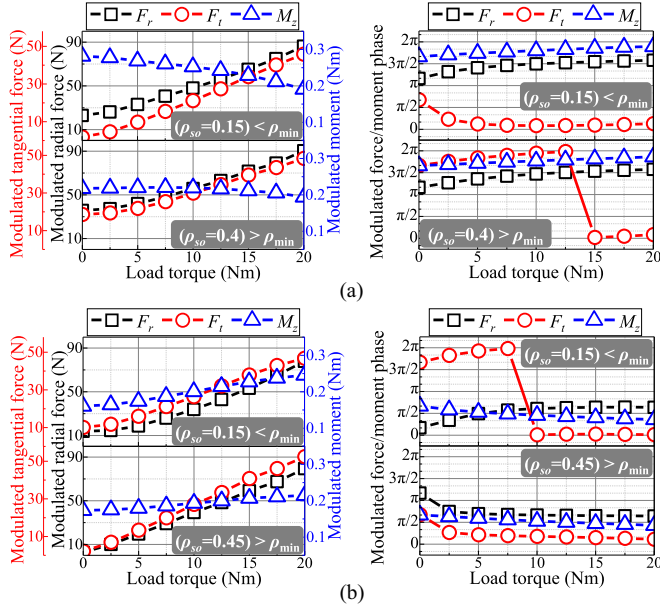


Fig. 5. Amplitude and phase of the lowest spatial order modulated excitations for $2p = S \pm 2$ FSCW PMSMs under different load conditions, for slot opening pitch regions $\rho_{so} < \rho_{min}$ and $\rho_{so} > \rho_{min}$. (a) 12S10P. (b) 12S14P.

TABLE III

PHASE OF THE LOWEST SPATIAL ORDER MODULATED EXCITATIONS OF $2P = S \pm 2$ UNDER LOAD CONDITION AT $2F_e$

Items	$2p = S - 2$		$2p = S + 2$	
	$\rho_{so} < \rho_{min}$	$\rho_{so} > \rho_{min}$	$\rho_{so} < \rho_{min}$	$\rho_{so} > \rho_{min}$
Modulated radial force	$\pi \sim 3\pi/2$	$\pi \sim 3\pi/2$	$0 \sim \pi/2$	$\pi \sim \pi/2$
Modulated tangential force	$\pi/2 \sim 0$	$3\pi/2 \sim 2\pi$	$3\pi/2 \sim 2\pi$	$\pi/2 \sim 0$
Modulated moment	$3\pi/2 \sim 2\pi$	$3\pi/2 \sim 2\pi$	$\pi/2 \sim 0$	$\pi/2 \sim 0$

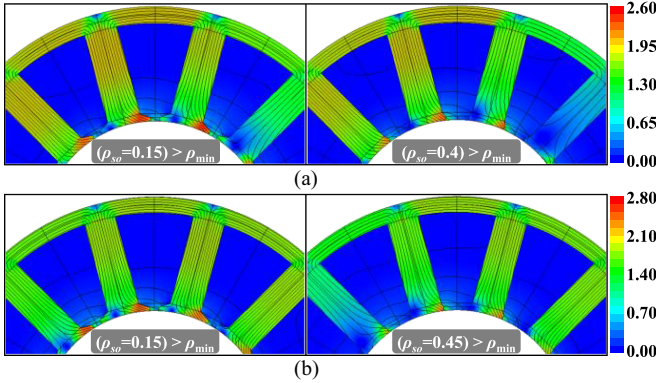


Fig. 6. Magnetic flux density contours for FSCW PMSMs with different slot opening pitch regions (Unit: T). (a) 12S10P. (b) 12S14P.

12S10P PMSM and slightly increases in the 12S14P PMSM. Fig. 6 shows the magnetic flux density contours obtained by FEA to illustrate the effect of the slot opening pitch on the magnetic field distribution. The slot opening pitch slightly modifies the local flux density near the tooth tips and slot openings.

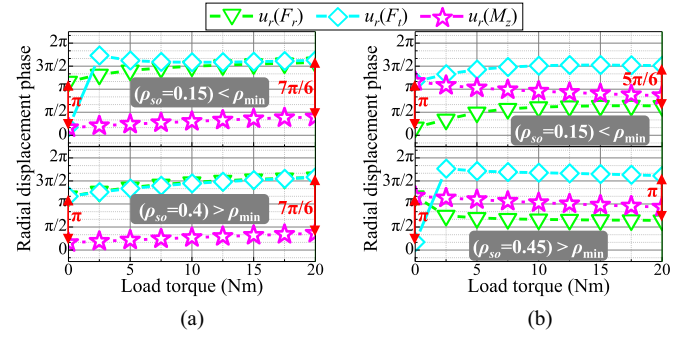


Fig. 7. Radial vibration phase induced by the lowest spatial order modulated excitations for $2p = S \pm 2$ FSCW PMSMs under different load conditions, for slot opening pitch regions $\rho_{so} < \rho_{min}$ and $\rho_{so} > \rho_{min}$. (a) 12S10P. (b) 12S14P.

III. EFFECT OF PHASE SHIFT OF MODULATED EM EXCITATIONS ON RADIAL VIBRATION

In this section, the influence of the phase shifts of the modulated excitations on the radial vibration of FSCW PMSMs is investigated. Based on the phase characteristics of the lowest spatial order modulated excitations derived and validated in Section II, the vibration contributions of each excitation component are analyzed by considering their relative phase relationships under no-load and load conditions for different slot opening pitches. According to [23], the phase of the resulting radial vibration is in phase with the modulated radial force, lags the phase of the modulated tangential force by $\pi/2$, and leads the phase of the modulated moment by $\pi/2$. This phase relationship originates from the structural response of the stator, where tangential force and moment induce shear and bending responses that introduce a $\pm\pi/2$ phase shift in the radial vibration. Therefore, the phase of the radial vibration induced by each excitation can be determined from the phase characteristics of the modulated excitations shown in Fig. 5. The corresponding radial vibration phases are calculated as shown in Fig. 7, where u_r denotes the radial displacement.

A. Radial Vibration Contributions

1). *No-Load Conditions:* The contribution of each modulated excitation to radial vibration is determined by the phase differences among the vibration components induced by the modulated excitations. Based on the superposition principle of structural vibration [23], the resultant vibration can be expressed as the vector sum of two harmonic components with the same frequency

$$X = A_1 e^{j\Phi_1} + A_2 e^{j\Phi_2}, \quad (A_1 \leq A_2) \quad (12)$$

where A and Φ are the amplitude and phase of each component, respectively. The phase difference satisfying $|X| = A_2$ can be obtained as

$$\Phi_1 - \Phi_2 = \pm \cos^{-1} \left(-\frac{A_1}{2A_2} \right) + 2\pi k. \quad (13)$$

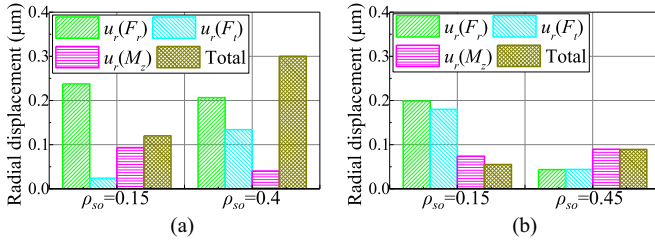


Fig. 8. Results of the pole-frequency vibration contributions by the modulated excitations under no-load condition. (a) 12S10P. (b) 12S14P.

TABLE IV

PHASE OF THE RADIAL VIBRATION BY MODULATED EXCITATIONS OF $2P = S \pm 2$ UNDER NO-LOAD CONDITION AT $2F_e$

Items	$2p = S - 2$		$2p = S + 2$	
	$\rho_{so} < \rho_{min}$	$\rho_{so} > \rho_{min}$	$\rho_{so} < \rho_{min}$	$\rho_{so} > \rho_{min}$
Modulated radial force	π	π	0	π
Modulated tangential force	0	π	π	0
Modulated moment	0	0	π	π

When $A_1 = A_2$, the condition $|X| = A_2$ is satisfied at $2\pi/3$ and $4\pi/3$. When $A_1 \ll A_2$, the corresponding phase difference approaches $\pi/2$ and $3\pi/2$. Therefore, regardless of the vibration amplitudes, the resultant amplitude is always suppressed when the phase difference lies within $2\pi/3$ to $4\pi/3$, whereas it is always enhanced when the phase difference lies within 0 to $\pi/2$ or $3\pi/2$ to 2π . As shown in Fig. 7, the vibration of the 12S10P PMSM under no-load conditions strongly depends on the phase of the modulated tangential force. As the slot opening pitch varies, the vibration induced by the modulated tangential force alternates between vibration enhancement and suppression regions. In contrast, the vibration from the modulated moment remains within the suppression region regardless of the slot opening pitch. For the 12S14P PMSM, the phase of the vibration from the modulated radial force changes, which alters the vibration contribution of the modulated moment. In contrast, the vibration from the modulated tangential force remains within the suppression region regardless of slot opening pitch region. To verify the radial vibration contributions of each modulated excitation under no-load conditions, 3-D structural FEA was performed. Each excitation was individually applied at the stator tooth tips of the structural model to evaluate its vibration contributions [17]. As shown in Fig. 8, the simulated radial vibration responses for both the 12S10P and 12S14P PMSMs agree well with the predicted vibration contributions over the investigated slot opening pitch. Accordingly, the phase relationships of the pole-frequency vibration from the modulated excitations under no-load conditions are given in Table IV.

2). Load Conditions: Under load conditions, the phase characteristics of the radial vibration induced by the lowest spatial order modulated excitations differ from those under no-load conditions. As shown in Fig. 7, the phase of the radial vibration from the modulated excitations shows a consistent tendency for each of the 12S10P and 12S14P PMSMs as the load torque increases, regardless of the slot opening pitch. For the 12S10P PMSM, the vibration induced by the modulated tangential force becomes

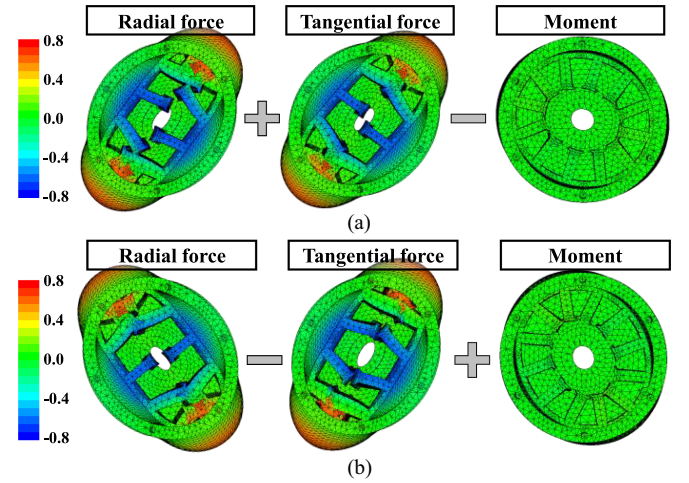


Fig. 9. Pole-frequency vibration contributions of $2p = S \pm 2$ FSCW PMSMs by the modulated excitations under load condition (unit: μm and scale: 4×10^4). (a) 12S10P. (b) 12S14P.

TABLE V

PHASE OF THE RADIAL VIBRATION BY MODULATED EXCITATIONS OF $2P = S \pm 2$ UNDER LOAD CONDITION AT $2F_e$

Items	$2p = S - 2$		$2p = S + 2$	
	$\rho_{so} < \rho_{min}$	$\rho_{so} > \rho_{min}$	$\rho_{so} < \rho_{min}$	$\rho_{so} > \rho_{min}$
Modulated radial force	$\pi \sim 3\pi/2$	$\pi \sim 3\pi/2$	$0 \sim \pi/2$	$\pi \sim \pi/2$
Modulated tangential force	$2\pi \sim 3\pi/2$	$\pi \sim 3\pi/2$	$\pi \sim 3\pi/2$	$2\pi \sim 3\pi/2$
Modulated moment	$0 \sim \pi/2$	$0 \sim \pi/2$	$\pi \sim \pi/2$	$\pi \sim \pi/2$

increasingly in phase with that from the modulated radial force, resulting in enhanced radial vibration. In contrast, the vibration from the modulated moment corresponds to vibration suppression. For the 12S14P PMSM, the vibration from the modulated tangential force corresponds to vibration suppression, whereas the vibration from the modulated moment corresponds to vibration enhancement. To verify these vibration contributions, 3-D structural FEA was conducted under a load torque of 20 N·m. The vibration contributions of the individual excitations for the 12S10P and 12S14P PMSMs are shown in Fig. 9. The simulation results agree well with the predicted vibration contributions, confirming the validity of the phase-based vibration analysis. Accordingly, the phase relationships of the vibration components induced by the modulated excitations under load conditions are given in Table V. As indicated in Fig. 5, the amplitudes of the modulated radial and tangential forces increase significantly with load torque, whereas the amplitude of the modulated moment remains nearly unchanged. Consequently, the influence of the modulated tangential force on radial vibration becomes dominant. Since the modulated tangential force acts as a suppressive excitation for radial vibration in the 12S14P PMSM, this configuration exhibits more favorable vibration characteristics under high-load conditions. Therefore, the 12S14P configuration is advantageous for radial vibration performance when large instantaneous load torques are required. This characteristic is beneficial for humanoid robot joint applications. Finally, the overall pole-frequency vibration contributions of the modulated

TABLE VI
POLE-FREQUENCY RADIAL VIBRATION CONTRIBUTIONS BY MODULATED EXCITATIONS OF $2p = S \pm 2$

Conditions	Items	$2p = S - 2$		$2p = S + 2$	
		$\rho_{so} < \rho_{min}$	$\rho_{so} > \rho_{min}$	$\rho_{so} < \rho_{min}$	$\rho_{so} > \rho_{min}$
No-load	F_r	Enhance	Enhance	Enhance	Enhance
	F_t	Suppress	Enhance	Suppress	Suppress
	M_z	Suppress	Suppress	Suppress	Enhance
Load	F_r	Enhance		Enhance	
	F_t	Enhance		Suppress	
	M_z	Suppress		Enhance	

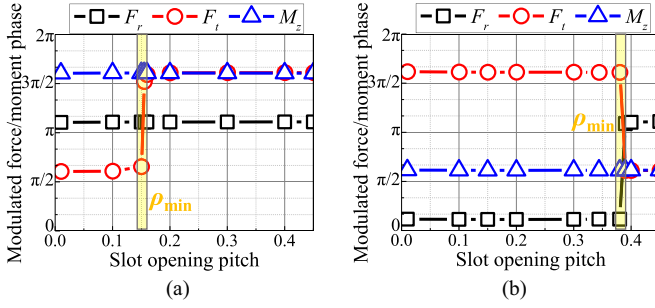


Fig. 10. Phase of the lowest spatial order modulated excitations for $2p = S \pm 2$ FSCW PMSMs under no-load condition, according to the slot opening pitch. (a) 18S16P. (b) 18S20P.

excitations in FSCW PMSMs with $2p = S \pm 2$ are given in Table VI.

B. Analysis of Different $2p = S \pm 2$ Pole-Slot Combinations

To examine the generality of the phase-dependent vibration characteristics, additional pole-slot combinations with $2p = S \pm 2$ are analyzed. Based on the specifications in Table II, 18S16P and 18S20P PMSM models are derived by adjusting the number of series turns per phase to satisfy the rated torque. Fig. 10 shows the phases of the lowest spatial order modulated excitations for the 18S16P and 18S20P PMSMs under no-load conditions as the slot opening pitch varies. Although ρ_{min} differs from that in Fig. 4, the phase transition behavior remains unchanged. Accordingly, the resulting radial vibration phase characteristics are consistent with Table IV. The phase characteristics under load conditions are further examined in Fig. 11. For both PMSMs, the phase tendencies of the lowest spatial order modulated excitations under different load torques and slot opening pitch regions are consistent with those shown in Fig. 5. Accordingly, the resulting radial vibration phase characteristics are consistent with Table V. Consequently, the 18S16P and 18S20P PMSMs also exhibit radial vibration contributions consistent with Table VI.

IV. EXPERIMENTAL VERIFICATION

To experimentally validate the radial vibration contributions for pole-slot combinations with $2p = S \pm 2$ and different slot opening pitches, vibration tests were conducted. Four prototypes were fabricated, including two 12S10P PMSMs and two 12S14P

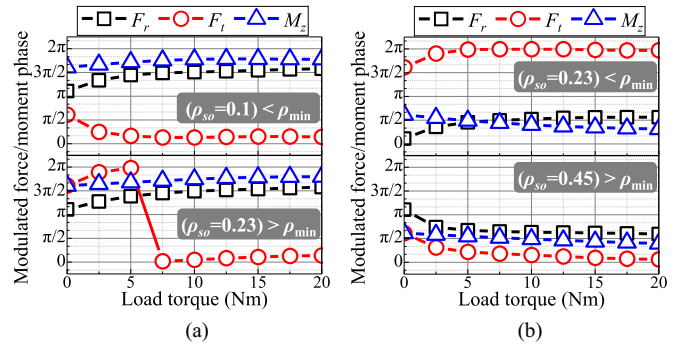


Fig. 11. Phase of the lowest spatial order modulated excitations for $2p = S \pm 2$ FSCW PMSMs under different load conditions, for slot opening pitch regions $\rho_{so} < \rho_{min}$ and $\rho_{so} > \rho_{min}$. (a) 18S16P. (b) 18S20P.

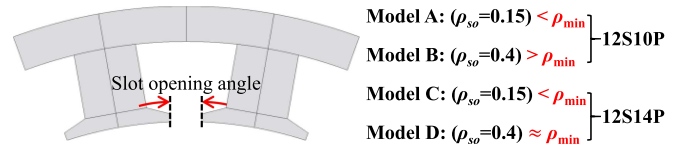


Fig. 12. Slot opening pitch and pole-slot combinations of the four prototype FSCW PMSMs.

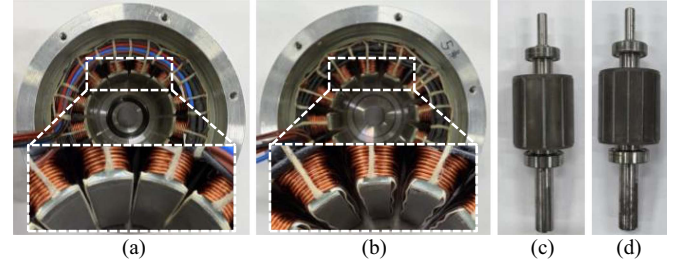


Fig. 13. Fabricated prototypes of 12S10P and 12S14P PMSMs with different slot opening pitch. (a) Stator with slot opening pitch of 0.15. (b) Stator with slot opening pitch of 0.4. (c) Rotor with 10-pole. (d) Rotor with 14-pole.

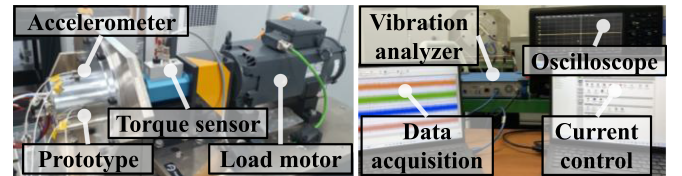


Fig. 14. Load and vibration test setup.

PMSMs, with specifications based on Table II. As illustrated in Fig. 12, Models A and B correspond to the 12S10P PMSMs with slot opening pitches of 0.15 and 0.4, respectively, while models C and D correspond to the 12S14P PMSMs with slot opening pitches of 0.15 and 0.4, respectively. As analyzed in Section II, Models A and C have slot opening pitches smaller than ρ_{min} , Model B has a slot opening pitch larger than ρ_{min} , and Model D has a slot opening pitch close to ρ_{min} . Fig. 13 shows the fabricated stator and rotor prototypes. Fig. 14 shows

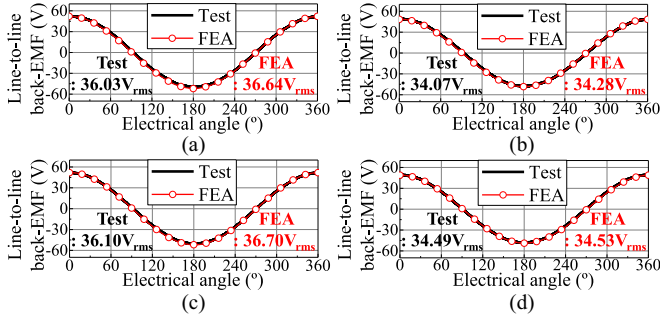


Fig. 15. Line-to-line back-EMF test and FEA results at 1000 r/min. (a) Model A. (b) Model B. (c) Model C. (d) Model D.

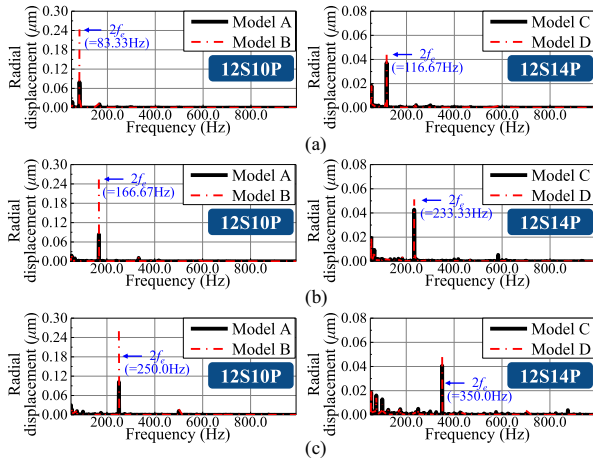


Fig. 16. Measured radial vibration responses of four prototypes under no-load conditions at different speeds. (a) 500 r/min. (b) 1000 r/min. (c) 1500 r/min.

the experimental setup used for the load and vibration tests. The prototypes were assembled with careful alignment to minimize assembly tolerance. To verify that the prototypes were properly manufactured, the back electro-motive force (back-EMF) was measured at 1000 r/min and compared with the results obtained from 2-D EM FEA. The measured back-EMF waveforms were processed to remove DC offset. As shown in Fig. 15, the measured back-EMF waveforms and RMS values of all prototypes agree well with the corresponding FEA results, confirming the validity of the fabricated motor models. The vibration tests were conducted at different speeds under no-load conditions and at 1000 r/min under rated load conditions. A Kistler 8776A50 accelerometer with a sensitivity of 100 mV/g was used to measure the vibration responses. Four accelerometers were mounted on the motor housing, where two were positioned at the slot centers and the other two at the tooth centers. The measured time domain acceleration signals were converted into the frequency domain using FFT analysis, and the averaged acceleration spectrum was obtained. A Hanning window was applied during the signal processing to mitigate spectral leakage. The radial displacement was then obtained by dividing the acceleration spectrum by the square of the corresponding angular frequency. Fig. 16 shows the measured radial vibration responses of the four prototype models under no-load conditions at different speeds. As the

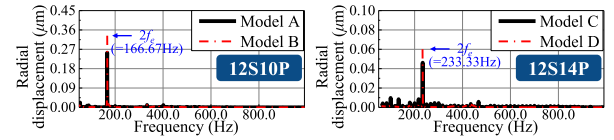


Fig. 17. Measured radial vibration responses of four prototypes under rated load conditions at 1000 r/min.

rotational speed increases, the frequencies of the pole-frequency modulated excitations scale proportionally with speed, while their phase relationships remain unchanged. Consequently, the resulting vibration characteristics are preserved. Model B exhibits significantly larger vibration than Model A due to both the larger amplitude of the lowest spatial order modulated radial force and the phase reversal of the modulated tangential force across $\rho_{\min}$, leading to vibration enhancement as given in Table VI. When comparing models C and D, model D shows slightly higher vibration levels. Although the modulated radial and tangential forces in model D are nearly zero, the modulated moment induces vibration in opposite phase to the modulated radial force. In contrast, the vibration in model C is partially suppressed by the opposing contributions of the modulated tangential force and moment despite the larger modulated radial force. Fig. 17 shows the measured radial vibration responses of the four models under rated load conditions. Unlike the no-load condition, phase-dependent vibration contributions associated with the pole-slot combination are observed regardless of the slot opening pitch. Specifically, in models C and D, the modulated tangential force suppresses the radial vibration, resulting in significantly lower vibration than in models A and B.

V. CONCLUSION

This article investigated the phase characteristics of the lowest spatial order modulated excitations and their influence on radial vibration in FSCW PMSMs with $2p = S \pm 2$ pole-slot combinations, which are widely used in robotic joint applications. First, the phase shifts of the modulated excitations were analytically derived by considering the tangential AEFD depending on the slot opening pitch and pole-slot combination. The relative phase relationships of the modulated excitations were systematically established under both no-load and load conditions using FEA. Next, the phase-dependent vibration contributions of the modulated excitations were analyzed. In particular, as the load increases, the $2p = S + 2$ combination exhibits more favorable vibration performance than the $2p = S - 2$ combination, since the modulated tangential force suppresses radial vibration. Furthermore, additional pole-slot combinations with $2p = S \pm 2$ were examined to verify the generality of the phase-dependent vibration contributions. Finally, vibration tests on four prototypes validated the predicted radial vibration contributions under both no-load and load conditions. These results elucidate the phase-dependent vibration mechanism and provide practical guidelines indicating that the $2p = S + 2$ combination is preferable for reducing radial vibration in robotic joint motors. However, the present study focuses on FSCW PMSMs with $2p = S \pm 2$ pole-slot combinations, and the

experimental verification was conducted under rated load conditions. Future work will extend the present analysis to investigate radial vibration reduction strategies for FSCW PMSMs with various pole–slot combinations.

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