Improved Frozen Permeability Method Considering Load Condition Flux Path for Dual Stator Permanent Magnet Synchronous Motor

Young-Jae Kang[1], Cheon-Ho Song[1], and Myung-Seop Lim[2]

[1]Department of Automotive Engineering (Automotive-Computer Convergence), Hanyang University, Seoul 04763, Republic of Korea

[2]Department of Automotive Engineering, Hanyang University, Seoul 04763, Republic of Korea

Urban air mobility (UAM) requires highly reliable propulsion systems, for which the dual stator permanent magnet synchronous motor (PMSM) is suitable owing to its structural redundancy. Accurate parameter estimation is essential for modeling such motors. This article proposes a frozen permeability method (FPM) that maintains the magnetic flux path under load conditions to improve the accuracy of parameter estimation. In this method, the permanent magnet (PM) is retained as a magnetic source during flux linkage calculation, and its internal magnetization direction is considered. To verify the effectiveness of the proposed method, the conventional FPM, which treats the PM as air, was defined as a comparison reference. Simulation results confirmed that the proposed method reduces torque estimation errors by maintaining the correct flux path under load conditions. Experimental validation was also conducted, confirming the applicability of the proposed method under various current amplitudes and phase angle conditions. The proposed method contributes to improving parameter estimation accuracy for dual stator PMSM, which is expected to enhance the modeling accuracy of motor systems in high-precision control and digital twin-based applications.

Index Terms— **Dual stator, frozen permeability method (FPM), magnetic flux path, parameter estimation.**

I. Introduction

URBAN air mobility (UAM) is being developed as a new urban transportation solution for carrying passengers and cargo through urban airspace [1]. UAM primarily adopts electric vertical take-off and landing (eVTOL) propulsion systems to overcome spatial constraints in urban areas, particularly the lack of runways. As these systems are operated in urban environments, a high level of reliability is required [2], [3]. The dual stator permanent magnet synchronous motor (PMSM) is well suited for eVTOL systems that require high reliability, as it enables both structural and electrical redundancy through two stators independently driven by separate inverters [4].

To ensure the high level of safety required for UAM, accurate performance prediction and control of the eVTOL propulsion system are essential. In particular, it is critical to accurately predict the performance of the propulsion motor, which generates thrust, and to precisely control its operation. Although finite element analysis (FEA) provides high accuracy in motor performance prediction, its high computational cost in terms of time and resources makes it unsuitable for real-time applications such as controllers or digital twins. Parameter-based models that utilize look-up tables generated from pre-simulated FEA data can reduce computational cost by lowering data volume [5]. These models are essential for controller design, as they can rapidly predict motor characteristics and performance under various operating conditions using parameters such as inductance and flux linkage. In particular, for the dual stator PMSM applied to eVTOL propulsion systems, inductance-based modeling is primarily utilized for control, and therefore accurate inductance estimation is required. Accurate inductance is also necessary for fault analysis [6], and for sensorless control to address potential sensor failures [7], both of which are required to ensure system safety.

To design and implement an accurate controller, precise estimation of motor parameters is essential. The accuracy of parameter estimation becomes even more critical in applications such as UAM, where small estimation errors can significantly affect vehicle stability. Common methods for motor parameter estimation include the incremental inductance method and the frozen permeability method (FPM). The incremental inductance method estimates inductance as the ratio of the change in flux linkage to a small variation in current [8]. While the incremental inductance method is faster, it cannot account for mutual components between the d- and q-axes. In contrast, the FPM involves a higher computational cost but can reflect these components. Therefore, the FPM is commonly used for accurate controller design. The FPM estimates inductance linearly based on the principle of superposition, by fixing the permeability of magnetic materials at each analysis step under load conditions and evaluating the effect of each magnetomotive force (MMF) source individually [9]. In some cases, the FPM has been slightly modified depending on the motor characteristics. In a PMSM using NdFeB magnets, the permeability of the magnet is nearly constant and close to that of air. For computational convenience, the magnet is treated as air in the analysis [10]. An FPM considering the hysteresis characteristics of permanent magnet (PM) has been studied [11]. In the case of

Received 19 July 2025; revised 25 September 2025; accepted 6 October 2025. Date of publication 14 October 2025; date of current version 13 July 2026. Corresponding author: M.-S. Lim (e-mail: myungseop@hanyang.ac.kr).

Color versions of one or more figures in this article are available at https://doi.org/10.1109/TMAG.2025.3620874.

Digital Object Identifier 10.1109/TMAG.2025.3620874

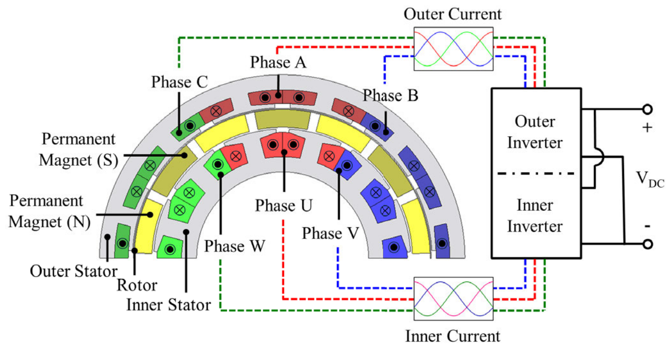

Fig. 1. Configuration of dual stator PMSM with dual inverters.

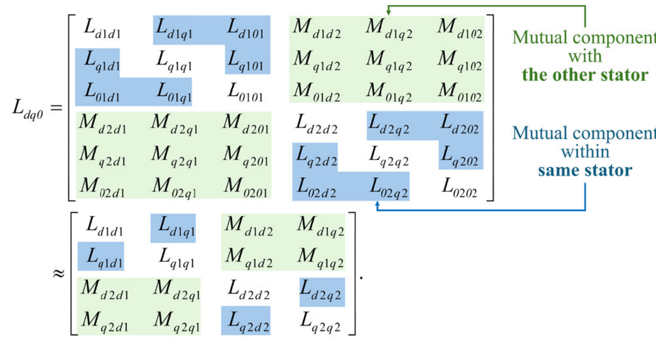

Fig. 2. Inductance matrix of dual stator PMSM.

variable flux PM machines, an approach that divides the PM into multiple elements to reflect the varying permeability of the magnet has been reported [12], [13]. For the dual stator PMSM, a modified parameter estimation approach has also been applied to improve accuracy. However, such a method has been limited to the incremental inductance method only [14].

Owing to the unique structural characteristics of the dual stator PMSM, a modification of the FPM is required to enable accurate parameter estimation. This article proposes the FPM that accurately reflects the flux path within the PM under load conditions in the dual stator PMSM. To validate the proposed method, the conventional FPM is defined and compared. Torque calculated using the parameters derived from each method is compared with results obtained from FEA. In addition, experimental validation confirms the applicability of the proposed method.

II. Dual Stator PMSM

The dual stator PMSM comprises an inner and an outer stator positioned along the radial direction, with a rotor positioned between them. Each stator generates a three-phase rotating magnetic field through independently controlled inverters. PM is attached to the rotor in alternating N and S poles. The magnetic fluxes generated by the rotating magnetic fields of the two stators interact as they pass through the rotor, resulting in torque generation. Fig. 1 illustrates the configuration of the reference dual stator PMSM used for analysis in this article, where the outer and inner three-phase currents are generally controlled with the same magnitude and phase to maximize torque and efficiency. In this structure, the magnetic fluxes generated by the inner and outer stators interact with each other, which requires the inclusion of mutual inductance terms between the stators in mathematical modeling. Fig. 2 shows the inductance matrix of the dual stator PMSM expressed in the d–q reference frame. The components highlighted in the blue box represent the mutual terms within the same stator, while the components highlighted in the green box indicate the mutual terms between the two stators.

Here, the subscripts d, q, and 0 denote the d-axis, q-axis, and zero-sequence components, respectively. The outer stator components are denoted by subscript 1, and the inner stator components are denoted by the subscript 2. The subscript notation follows the rule that in L_{ab} or M_{ab}, the subscript ab indicates that the flux produced in winding b is linked to winding a. L represents the components generated within the same stator, while M denotes the mutual components resulting from the interaction between the two stators. Unlike a conventional three-phase motor, where the inductance matrix has a 2×2 form, the dual stator PMSM requires an expanded 4×4 matrix, which includes the mutual components between inner and outer stators. In this article, a balanced three-phase current with a Y-connection is assumed, and therefore, the zero-sequence component is neglected. The d–q axis flux linkage λ_{dq} can be expressed as follows using the inductance matrix:

$$\lambda_{\text{dq}} = \begin{bmatrix} \lambda_{d1} \\ \lambda_{q1} \\ \lambda_{d2} \\ \lambda_{q2} \end{bmatrix} = L_{\text{dq}} \times \begin{bmatrix} i_{d1} \\ i_{q1} \\ i_{d2} \\ i_{q2} \end{bmatrix} + \begin{bmatrix} \Psi_{\text{fd1}} \\ \Psi_{\text{fq1}} \\ \Psi_{\text{fd2}} \\ \Psi_{\text{fq2}} \end{bmatrix}. \quad (1)$$

Here, λ_d and λ_q represent d-axis and q-axis flux linkage, i_d, i_q, and Ψ represent the d-axis and q-axis armature currents, and the field flux linkage generated by the PM. Based on this flux linkage, the induced voltage V_o can be expressed as follows:

$$V_{o1,2} = \sqrt{V_{\text{od1,2}}^2 + V_{\text{oq1,2}}^2} = \omega\sqrt{\lambda_{d1,2}^2 + \lambda_{q1,2}^2}. \quad (2)$$

Here, V_{od} and V_{oq} represent the d-axis and q-axis induced voltages, respectively. ω denotes the electrical angular speed. Finally, the following equation represents the torque T:

$$\begin{aligned} T &= \frac{V_{\text{od1}} i_{d1} + V_{\text{oq1}} i_{q1} + V_{\text{od2}} i_{d2} + V_{\text{oq2}} i_{q2}}{\omega} \\ &= \frac{3}{2}\text{pp}\big(\lambda_{q1} i_{d1} + \lambda_{d1} i_{q1} + \lambda_{d2} i_{q2} + \lambda_{q2} i_{d2}\big). \end{aligned} \quad (3)$$

Here, pp represents the pole pair.

III. Frozen Permeability Method

This section defines the conventional FPM and compares it with the proposed method in this article for the dual stator PMSM. Owing to the structural characteristics of the dual stator PMSM, the PM is located at the center of the flux path that crosses the two air gaps between the inner and outer stators. This positional characteristic of the PM requires accurate consideration of the magnetization direction of the PM in magnetic analysis for parameter estimation. Inaccurate reproduction of the flux path in the PM under load conditions leads to inaccurate prediction of the flux linkage. This is particularly critical for the mutual flux between the two stators.

A. Conventional Method

The FPM estimates parameters linearly by separating the total flux linkage under load conditions into components attributed to each MMF source. The method consists of three main steps: first, the permeability of each element comprising the core is fixed at each simulation step under load conditions. Second, each MMF source is activated individually to calculate the phase flux linkage attributed to that source. Finally, the phase flux linkages obtained from each MMF source are used to calculate the phase inductances. The obtained phase inductances and phase flux linkages are then transformed into the d-q reference frame by applying the Clarke–Park transformation $[T]$, as shown in the following equations:

$$T(\theta) = \frac{2}{3}\begin{bmatrix} \cos\theta & \cos\left(\theta - \frac{2\pi}{3}\right) & \cos\left(\theta + \frac{2\pi}{3}\right) \\ -\sin\theta & -\sin\left(\theta - \frac{2\pi}{3}\right) & -\sin\left(\theta + \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \quad (4)$$

$$[T] = \begin{bmatrix} T(\theta_1) & 0 \\ 0 & T(\theta_2) \end{bmatrix}. \quad (5)$$

Here, θ represents electrical angle. This approach enables linear analysis of nonlinear magnetic behavior by fixing the permeability of each element of the core, allowing the effects of each MMF source to be analyzed based on the principle of superposition. Several approaches based on the FPM have been proposed, in which the permeability of the PM is treated differently depending on motor characteristics. In this article, the conventional method is defined as the simplest form of the FPM, in which the PM is treated as air based on the assumption that the permeability of the NdFeB magnet is close to that of air, as shown in Fig. 3. However, applying the conventional method to the dual stator PMSM results in errors reaching several percent. These errors are primarily caused by treating the PM, which is located at the center of the internal flux path in the dual stator structure, as air when deriving the phase flux linkage. Even if the permeability inside the PM is fixed uniformly, the magnetization direction is not properly considered, which leads to distortion of the flux path in that region.

B. Proposed Method

The procedure of the proposed method for the dual stator PMSM is illustrated in Fig. 4. The key difference from the conventional method is that the PM is retained as a magnetic source during the calculation of the phase flux linkage for each MMF source. The proposed method preserves the internal flux path within the PM, located between the air gaps, under load conditions. This is applied during the calculation of the phase flux linkage for each MMF source, ensuring that the flux is distributed according to the internal magnetization direction of the PM. Additionally, since the PM is included, an additional step is required to determine the phase inductance, in which the field flux linkage is subtracted from the total flux linkage obtained for each phase under unit current excitation, as expressed in the following equation:

$$\lambda_{\rm ph} = \lambda_{\alpha\beta} - \lambda_{\beta}^{f}, \quad \alpha,\ \beta \in \{a, b, c, u, v, w\} \quad (6)$$

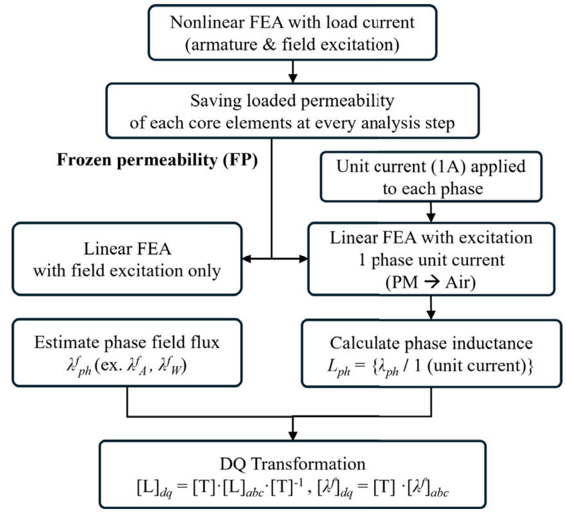

Fig. 3. Flow diagram of conventional FPM.

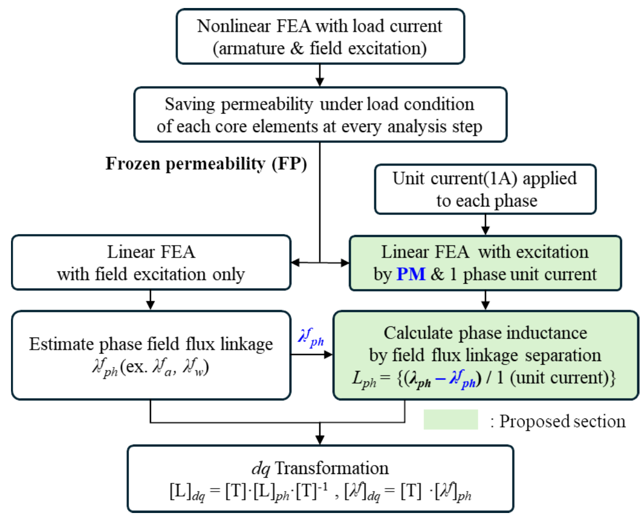

Fig. 4. Flow diagram of proposed FPM.

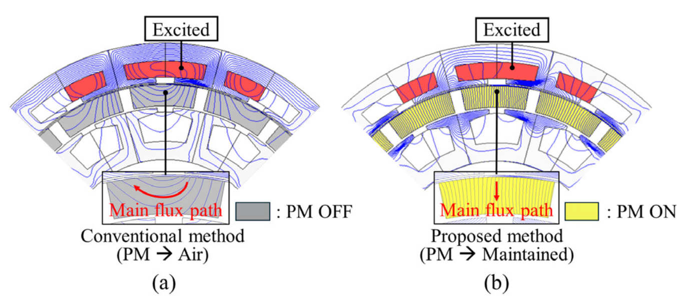

Fig. 5. Flux path comparison according to each FPM. (a) Conventional method. (b) Proposed method.

where $\lambda_{\rm ph}$ represents the phase flux linkage, α, β indicates that the flux generated by phase β is linked to phase α, and λ_{β}^{f} represents the field flux linkage linked to phase β. The terms inside the braces denote the three phases of each armature set.

C. Comparison of the Conventional and Proposed Methods

The conventional and proposed methods differ in how the PM, located at the center of the flux path, is treated when

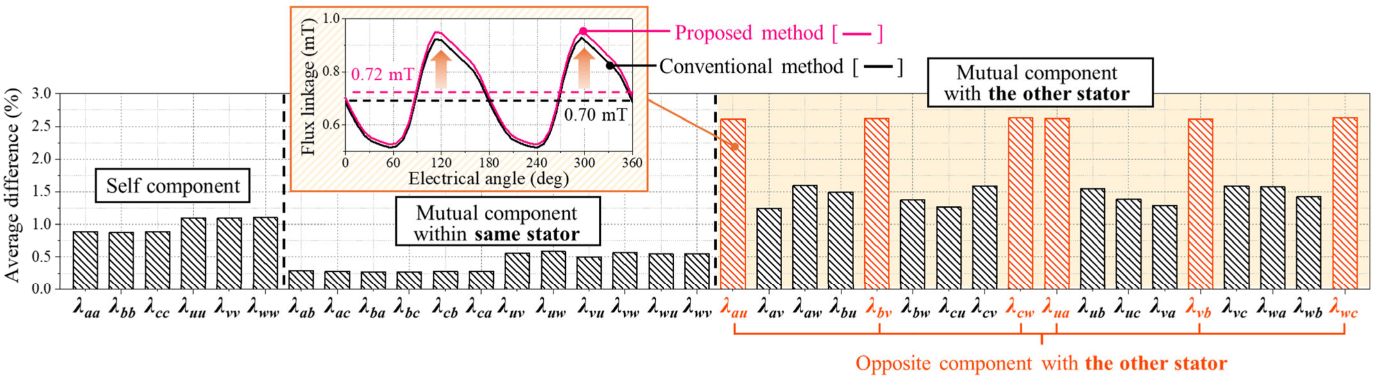

Fig. 6. Phase flux linkage difference between the conventional method and proposed method.

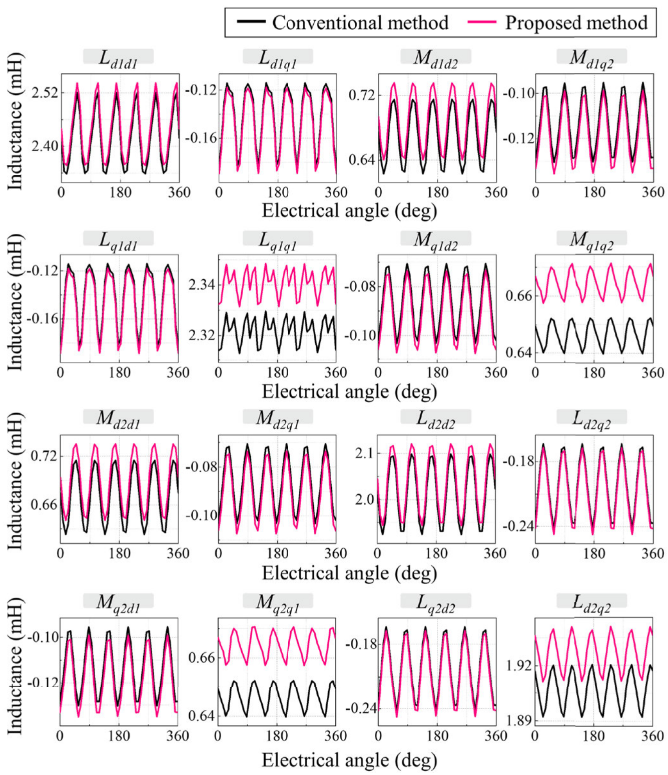

Fig. 7. Comparison of each inductance component calculated by each FPM.

applying the FPM to the dual stator PMSM. Fig. 5 shows the difference in flux paths between the conventional method and the proposed method when a unit current is applied. Fig. 5(a) shows the flux path within the PM obtained using the conventional method. In this case, the magnetic characteristics of the PM are not reflected, and as a result, a distorted flux path was observed. In contrast, as shown in Fig. 5(b), the proposed method preserves the magnetization direction inside the PM, and as a result, the flux path under load conditions was maintained. It was confirmed that the flux generated by the armature coil of the outer stator was properly transferred to the inner stator.

The difference in the flux paths between the conventional and proposed methods affects the calculation of the phase flux linkage. Fig. 6 shows the difference in phase flux linkage calculated by the conventional and proposed methods.

TABLE I
Average Inductance Comparison Between Conventional and Proposed FPM

Item	Conventional [mH]	Proposed [mH]	Difference [%]
L_{d1d1}	2.419	2.439	0.82
L_{d1q1}	-0.140	-0.150	2.92
M_{d1d2}	0.671	0.690	2.71
M_{d1q2}	-0.110	-0.120	3.65
L_{q1d1}	-0.140	-0.150	2.91
L_{q1q1}	2.322	2.341	0.81
M_{q1d2}	-0.087	-0.090	4.10
M_{q1q2}	0.647	0.664	2.68
M_{d2d1}	0.671	0.690	2.72
M_{d2q1}	-0.087	-0.090	4.06
L_{d2d2}	2.015	2.036	1.00
L_{d2q2}	-0.200	-0.210	1.96
M_{q2d1}	-0.110	-0.120	3.65
M_{q2q1}	0.647	0.664	2.67
L_{q2d2}	-0.200	-0.210	1.99
L_{q2q2}	1.907	1.927	1.02

The result was obtained under a load condition of 50 A_{rms} with a current phase angle of 0° ($i_d = 0$). A significant difference was observed in the mutual components between the stators.

Fig. 7 shows the d–q axis inductance calculated based on the phase flux linkage derived from the conventional and proposed methods. Due to the difference in the flux path, the conventional method resulted in high mutual components between the d- and q-axes. In contrast, the proposed method maintained the flux path through the PM, resulting in higher components along the same d-axis or q-axis between the two stators. The difference between the conventional and proposed methods is apparent in the q-axis components. The average values of the inductance are presented in Table I, where a relatively large difference is observed in the mutual components between the inner and outer stators.

The analysis results demonstrate that the proposed method accurately represents the mutual flux linkage between stators and indicates that the primary source of estimation errors in conventional method was this component. At the same time, since a single stator PMSM does not include such a component, the proposed method remains applicable to a

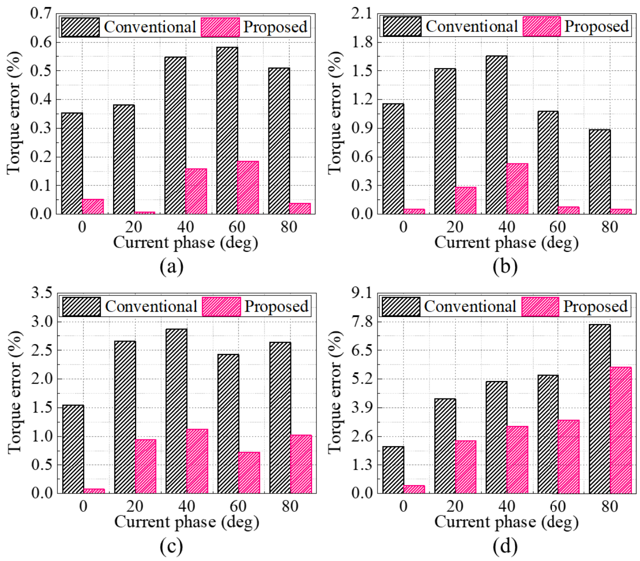

Fig. 8. Torque error comparison according to each FPM. (a) 50 A_{rms}. (b) 100 A_{rms}. (c) 150 A_{rms}. (d) 200 A_{rms}.

TABLE II
TORQUE COMPARISON ACCORDING TO EACH FPM

Load current [A_{rms}]	Phase angle [deg]	Conventional [Nm]	Proposed [Nm]	FEA [Nm]
50	0	211.9	211.0	211.2
	20	202.1	201.3	201.3
	40	162.2	161.6	161.3
	60	101.2	100.8	100.6
	80	34.2	34.0	34.0
100	0	323.3	319.4	319.6
	20	310.4	306.6	305.7
	40	260.1	257.2	255.8
	60	174.4	172.7	172.6
	80	60.6	61.1	61.1
150	0	397.1	390.8	391.1
	20	378.1	371.8	368.3
	40	317.9	312.5	309.0
	60	216.7	213.1	211.6
	80	78.4	77.1	76.4
200	0	462.4	454.3	452.6
	20	441.7	433.6	423.5
	40	370.0	362.8	352.1
	60	251.7	246.9	238.9
	80	92.6	90.9	86.0

single stator PMSM, but the effectiveness of the proposed method may be limited.

The error between the torque calculated using the parameters from each method and the torque obtained from FEA is shown in Fig. 8. The torque calculated using the proposed method showed a smaller error compared to the FEA result, and the numerical comparison is presented in Table II. The conventional method exhibited a maximum torque error of 18.26 Nm compared to the FEA result. At 200 A_{rms} and 0°, the proposed method reduced the torque error from 9.71 to 1.62 Nm. The comparison results demonstrate that the proposed method provides more accurate parameter estimation than the conventional method.

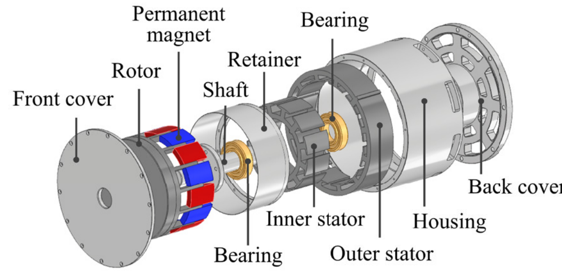

Fig. 9. Structure of the experimental dual stator PMSM.

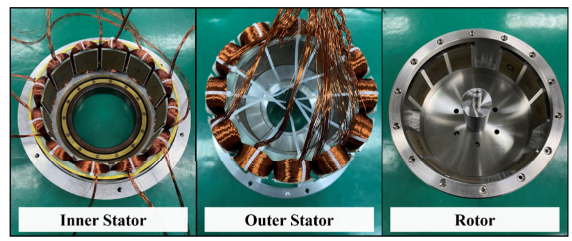

Fig. 10. Fabricated stators and rotor.

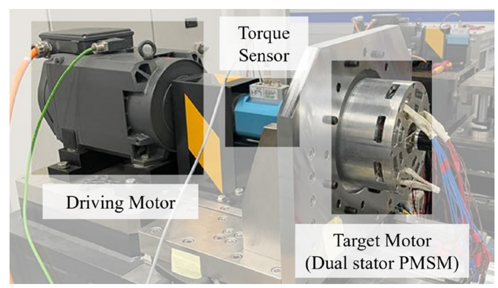

Fig. 11. Experimental setup.

IV. VERIFICATION

This section presents the experimental verification of the applicability of the proposed method. Figs. 9 and 10 show the structure of the prototype dual stator PMSM, which was a scaled-down model of the analysis motor, and Fig. 11 illustrates the experimental setup.

Ideally, experiments comparing the conventional and proposed methods would be performed under the same load conditions as those analyzed in the preceding FEA. However, due to safety limitations of the experimental setup, the maximum current was restricted to 5 A_{rms}. Therefore, the experimental validation focuses solely on confirming the accuracy of the proposed method by comparing the predicted torque with the measured torque under limited current conditions.

The experiment was conducted with current amplitudes of 1, 2, 3, 4, and 5 A_{rms}, and for each condition, current phase angles of 0°, 20°, 40°, 60°, and 80° were applied. Under the experimental conditions, the core materials operated in the nonlinear region. The experiment was conducted at 100 rpm, and the same current amplitude and phase angle were applied to both the inner and outer armature coils. The experimental results are presented in Fig. 12, which confirm that the predicted torque agrees well with the measured torque under various current amplitudes and phase angle conditions.

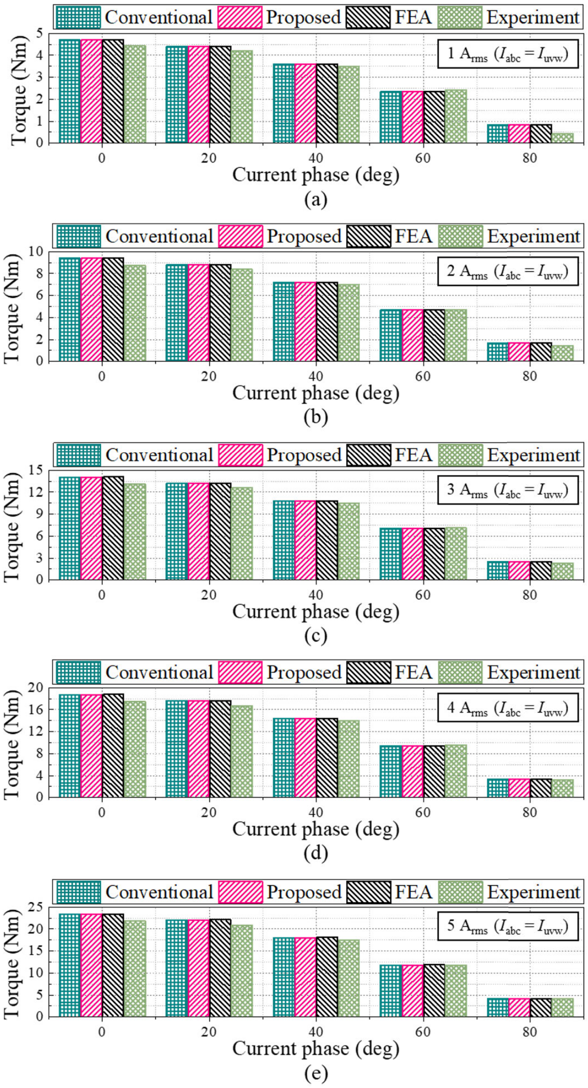

Fig. 12. Experimental results. (a) 1 A_{rms}. (b) 2 A_{rms}. (c) 3 A_{rms}. (d) 4 A_{rms}. (e) 5 A_{rms}.

V. Conclusion

This article proposes an FPM that considers the magnetization direction inside the PM to improve the accuracy of parameter estimation for a dual stator PMSM. In a dual stator PMSM, the PM is located between the two air gaps where torque is generated. Therefore, precise consideration of the internal magnetization direction of the PM is required. However, the conventional FPM treats the PM as air and thus cannot reflect the magnetization direction when calculating the phase flux linkage, leading to estimation errors in accurately modeling the flux interactions between the stators. To address this limitation, the proposed method retains the PM as a magnetic source during flux linkage calculation and reflects its magnetization direction while additionally separating the field flux component. Simulation results showed that the proposed method preserved the flux path under load conditions and enabled more accurate parameter estimation. The derived parameters were compared with torque results obtained from FEA, and the proposed method exhibited lower estimation errors than the conventional method. Experimental results demonstrated the applicability of the proposed method under various current amplitudes and phase angle conditions. In conclusion, the proposed FPM reflects the flux path more accurately by considering the magnetization direction of the PM, thereby improving the accuracy of parameter estimation for the dual stator PMSM. This improvement is expected to contribute to enhancing the modeling accuracy of motor systems in high-precision control and digital twin-based applications.

Acknowledgment

This work was supported by the National Research Foundation of Korea (NRF) grant funded by the Korea Government (MSIT) under Grant RS-2023-00207865.

References

[1] A. Cohen, S. Shaheen, and E. Farrar, "Urban air mobility: History, ecosystem, market potential, and challenges," in *Proc. IEEE/AIAA 38th Digit. Avionics Syst. Conf. (DASC).*, vol. 22, Sep. 2021, pp. 6074–6087.

[2] A. P. Thurlbeck and Y. Cao, "A mission profile-based reliability modeling framework for fault-tolerant electric propulsion," *IEEE Trans. Ind. Appl.*, vol. 58, no. 2, pp. 2312–2323, Mar. 2022.

[3] M. Graydon, N. Neogi, and K. S. Wasson, "Guidance for designing safety into urban air mobility: Hazard analysis techniques," in *Proc. AIAA Scitech*, Jul. 2020, p. 2099.

[4] S.-W. Hwang, D.-K. Son, S.-H. Park, G.-H. Lee, Y.-D. Yoon, and M.-S. Lim, "Design and analysis of dual stator PMSM with separately controlled dual three-phase winding for eVTOL propulsion," *IEEE Trans. Transport. Electrific.*, vol. 8, no. 4, pp. 4255–4264, Dec. 2022.

[5] A. J. P. Ortega, S. Das, R. Islam, and M. B. Kouhshahi, "High-fidelity analysis with multiphysics simulation for performance evaluation of electric motors used in traction applications," *IEEE Trans. Ind. Appl.*, vol. 59, no. 2, pp. 1273–1282, Mar. 2023.

[6] P. Arumugam, T. Hamiti, and C. Gerada, "Modeling of different winding configurations for fault-tolerant permanent magnet machines to restrain interturn short-circuit current," *IEEE Trans. Energy Convers.*, vol. 27, no. 2, pp. 351–361, Jun. 2012.

[7] M. Hinkkanen, S. E. Saarakkala, H. A. A. Awan, E. Mölsä, and T. Tuovinen, "Observers for sensorless synchronous motor drives: Framework for design and analysis," *IEEE Trans. Ind. Appl.*, vol. 54, no. 6, pp. 6090–6100, Nov. 2018.

[8] T. W. Nehl, F. A. Fouad, and N. A. Demerdash, "Determination of saturated values of rotating machinery incremental and apparent inductances by an energy perturbation method," *IEEE Power Eng. Rev.*, vols. PER–2, no. 12, pp. 28–29, Dec. 1982.

[9] J. A. Walker, D. G. Dorrell, and C. Cossar, "Flux-linkage calculation in permanent-magnet motors using the frozen permeabilities method," *IEEE Trans. Magn.*, vol. 41, no. 10, pp. 3946–3948, Oct. 2005.

[10] W. Q. Chu and Z. Q. Zhu, "Inductance calculation in variable-flux flux-intensifying permanent magnet synchronous machines using improved frozen permeability method," *IEEE Trans. Magnetics.*, vol. 49, no. 3, pp. 1202–1210, Mar. 2013.

[11] D. Shuto, Y. Takahashi, and K. Fujiwara, "Frozen permeability method for magnetic field analysis of permanent magnet motors considering hysteretic property," *IEEE Trans. Magn.*, vol. 55, no. 6, pp. 1–4, Jun. 2019.

[12] J. Chen, J. Li, R. Qu, and M. Ge, "Magnet-frozen-permeability FEA and DC-biased measurement for machine inductance: Application on a variable-flux PM machine," *IEEE Trans. Ind. Electron.*, vol. 65, no. 6, pp. 4599–4607, Jun. 2018.

[13] M. Ge, J. Li, R. Qu, Y. Lu, and J. Chen, "A synthetic frozen permeability method for torque separation in hybrid PM variable-flux machines," *IEEE Trans. Appl. Supercond.*, vol. 28, no. 3, pp. 1–5, Apr. 2018.

[14] C.-H. Song, J.-H. Lee, and M.-S. Lim, "Parameter estimation method for dual-stator permanent magnet synchronous motor based on incremental inductance," *IEEE Trans. Magn.*, vol. 60, no. 9, pp. 1–5, Sep. 2024.