Application of Response Surface Methodology to Robust Design of BLDC Motor

Young-Kyoun Kim, Jae-Ok Jo, Jung-Pyo Hong and Jin Hur

Abstract - This paper presents an approach to robust design of the brushless dc motor (BLDC motor) in which not only the cogging torque is reduced but also the robustness is enhanced. The approach is based on the response surface methodology (RSM) and the estimated model is used to minimize the total sensitivity of design variables. This approach is verified by the comparison of the robust solution with the normal result obtained by a conventional optimization procedure.

Keywaords – robust design, BLDC motor, RSM

1. Instruction

A design problem is generally a natural process to optimize the solution corresponding to specified requirements. The problem can be complex because there are many numbers of design variables and these design variables frequently interact with each other [1]. Moreover, when a motor is designed by using a conventional optimization algorithm, its performance cannot be satisfying in some cases. It is due to the limitations on the manufacturing tolerances, job requirements that cannot be presumed, adjustments for improving the efficiency of manufacture. In general, the variation in manufacturing processes can affect the machine performance, in terms of operating efficiency, reliability, and production of vibration. Recently, the demand for BLDC motors is expanding rapidly, and better good quality is required in some of the industrial applications. So, BLDC motors have to be designed in order to design specifications.

The cogging torque of BLDC motors arises from the interaction between its rotor magnet and slotted stator [2]. It exerts a bad influence on the motor performance. Therefore, this paper illustrates a robust design to reduce the cogging torque by using the RSM. The optimization technique based on the RSM is applied to the robust design of BLDC motor performance. The robust design is achieved by minimization of the total sensitivity concerning design variables.

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The RSM is well adapted to make an analytical model for a complex problem. Moreover, the RSM provides the designer with an overall perspective of the system response to the behavior of design variables within a design space [3, 4]. It can lead to great savings of time and efficiency without large repetition and expensive of computations. In this paper, the computation of the total sensitivity concerning design variables is done by using analytical model obtained from the RSM.

2. Method of Analysis

2.1 Define Design Variables

The cogging torque in the BLDC motor is produced by stator teeth interacting with the rotor-mounted permanent magnet. Therefore, three design variables are considered for the reduction of the cogging torque such as slot opening, stator tooth notching and dead zone of magnet pole. The analysis model of 6-pole, 18-slot BLDC motor and three design variables are shown in Fig. 1.

In this paper, it is assumed that the stator slot is skewed by a half of slot pitch for reducing the cogging torque and in order to consider slot skewing in two dimensions, a set of unskewed models cut by planes perpendicular to the shaft, is used as shown in Fig. 2.

2.2 Field Computation Method

Two-dimensional Finite Element Analysis (2-D FEA) is used to electromagnetic field. When making assumption as quasi-static field, displacement current can be neglected. Therefore, the electromagnetic governing equation on quasi-static field problem with field variables $\boldsymbol{A}$ is obtained by Maxwell's electromagnetic equation and as follows:

$$\nabla \times \left(\frac{1}{\mu} \nabla \times \boldsymbol{A}\right) = \boldsymbol{J}_0 + \boldsymbol{J}_m \tag{1}$$

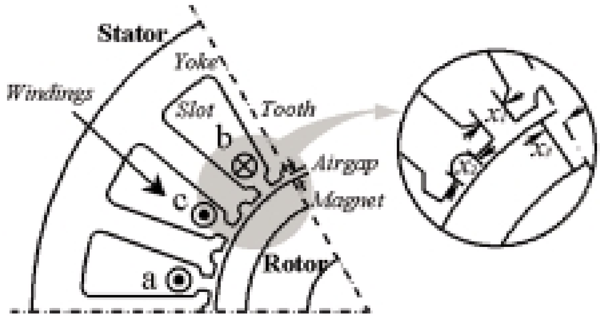

Fig. 1 Analysis model and design variables

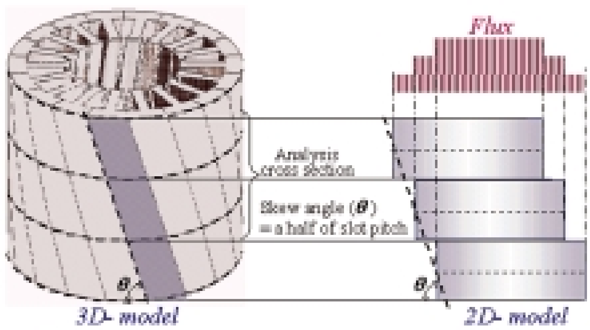

Fig. 2 Concept of the Skew model

where, $\boldsymbol{J_0}$ is the applied current density and $\boldsymbol{J_m}$ is the equivalent magnetizing current density, $\boldsymbol{A}$ is the magnetic vector potential, μ is the magnetic permeability. From Applying Coulomb gauge condition, the equation (1) can be rewritten in two-dimensional Cartesian coordinate and shown in (2). Therefore, $\boldsymbol{A}$ and $\boldsymbol{J_0}$, $\boldsymbol{J_m}$ have only $\boldsymbol{z}$ components.

$$\frac{1}{\mu}\left[\frac{\partial^2 \boldsymbol{A}}{\partial^2 \boldsymbol{x}^2}+\frac{\partial^2 \boldsymbol{A}}{\partial^2 \boldsymbol{y}^2}\right]+\boldsymbol{J}_0+\boldsymbol{J}_{\boldsymbol{m}}=0 \tag{2}$$

Equation (2) is the characteristic equation to analyze characteristic for the BLDC motor as 2-D FEM.

2.3 Calculation of Torque

Torque can be computed from the 2-D FEA results. Torque is calculated by Maxwell Stress Tensor method as the following (3). Equation (3) is obtained by the surface integration of a stress tensor vector $\boldsymbol{P}$ over an air gap enclosing the rotor surface as follows:

$$\boldsymbol{T}=\oint_s \boldsymbol{r}\times\boldsymbol{P}\,\mathrm{d}\boldsymbol{S} \tag{3}$$

where, $\boldsymbol{r}$ is distance vector of a point to axis rotation. Maxwell stress tensor is given by (4)

$$\boldsymbol{P}=\frac{1}{\mu_0}(\boldsymbol{n}\cdot\boldsymbol{B})\,\boldsymbol{B}-\frac{1}{2\mu_0}B^2\boldsymbol{n} \tag{4}$$

where, μ_0 is the permeability of free space, $\boldsymbol{n}$ is the normal vector to the surface $\boldsymbol{S}$, B is the magnetic flux density.
vector to the surface $\boldsymbol{S}$, B is the magnetic flux density.

2.4 Concept of The Response Surface Methodology

The RSM seeks to find the relationship between design variable and response through statistical fitting method, which is based on the observed data from the process or system. The response is generally obtained from real experiments or computer simulations, accordingly 2-D FEM is performed in this paper. It is supposed that the true response η can be written as follows [3]-[5]:

$$\eta=F(\zeta_1,\zeta_2,\cdots\zeta_k) \tag{5}$$

where, the variables ζ_1, ζ_2,…, ζ_k in (1) are expressed in natural units of a measurement, so called the natural variables. Because the form of the true response function F is unknown and perhaps very complicated, we must approximate it. In many cases, the approximating function y of the true response function F is normally chosen to be either a first-order or a second-order polynomial model, which is based on Taylor series expansion. In order to predict a curvature response, the second-order model is used at this paper. The relation between approximating function y and true response function F may be written as follows:

$$y=\beta_0+\sum_{j=1}^{k}\beta_j x_j+\sum_{j=1}^{k}\beta_{jj}x_j^2+\sum_{i\neq j}^{k}\beta_{ij}x_i x_j+\varepsilon \tag{6}$$

where, β is regression coefficients, ε denotes the random error and we treat ε as a statistical error, often assuming it to have a normal distribution with mean zero and variance σ^2. The observation response vector $\mathbf{y}$ at n data point of function y may be written in matrix notation as follows:

$$\mathbf{y}=\mathbf{X}\boldsymbol{\beta}+\boldsymbol{\varepsilon} \tag{7}$$

where, $\mathbf{X}$ is a matrix of the levels of the independent variables, $\boldsymbol{\beta}$ is a vector of the regression coefficients, $\boldsymbol{\varepsilon}$ is a vector of random error.

The least squares method, which is to minimize the sum of the squares of the random errors, is used to estimate unknown vector $\boldsymbol{\beta}$. The least squares function is as follows:

$$L=\sum_{i=1}^{n}\varepsilon_i^2=\boldsymbol{\varepsilon}'\boldsymbol{\varepsilon}=(\mathbf{y}-\mathbf{X}\boldsymbol{\beta})'(\mathbf{y}-\mathbf{X}\boldsymbol{\beta}) \tag{8}$$

The estimated vector $\mathbf{b}$ of the unknown vector $\boldsymbol{\beta}$ must satisfy as (9).

$$\left.\frac{\partial L}{\partial\boldsymbol{\beta}}\right|_{\mathbf{b}}=-2\mathbf{X}'\mathbf{y}+2\mathbf{X}'\mathbf{X}\mathbf{b}=0 \tag{9}$$

Therefore, the estimated vector $\mathbf{b}$ can be written as (10) and the fitted response vector $\hat{\mathbf{y}}$ is given by (11).

$$\mathbf{b}=(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y} \tag{10}$$

$$\hat{\mathbf{y}}=\mathbf{X}\mathbf{b} \tag{11}$$

where, $\mathbf{X}'$ is the transpose of the matrix **X**.

3. Optimization and Robust Design

The general formation of a conventional optimization is expressed as following [6].

$$\text{Minimize: } f(x_1, x_2, \cdots, x_k) \tag{12}$$

$$\text{Subject to: } g_i(x_1, x_2, \cdots, x_k) \le 0, \quad i = 1, 2, \cdots, m \tag{13}$$

$$x_{iL} \le x_i \le x_{iU} \qquad i = 1, 2, \cdots, k \tag{14}$$

where, $f(x_1,x_2,\ldots,x_k)$ is the objective function, $g_i(x_1, x_2,\ldots,x_k)$ is the constraint functions with the dimension of m, x_{iL} and x_{iU} is lower and upper bounds of design variables x_i and k is the total number of design variables respectively. The goal of the optimization is to reduce the cogging torque and to satisfy of the running torque. The analytical model built from the RSM can be use as either objective functions or constraint functions in an optimization procedure. The analytical model of the cogging torque is used as the objection function and that of the running torque is used as the constraint function respectively.

The next step in the design is to minimize the total sensitivity of design variables since the variation of system response makes the performance to be unstable. In order to obtain the robust optimal solution, the new objective function is defined as follows:

$$\text{Minimize: } \varphi(x) = \sum_{i=1}^{k} \frac{\partial f(x_1, x_2, \cdots, x_k)}{\partial x_i} + (f - f_0)^2 \tag{15}$$

$$\text{Subject to: } g_i(x_1, x_2, \cdots, x_k) \le 0, \quad i = 1, 2, \cdots, m \tag{16}$$

$$x_{iL} \le x_i \le x_{iU} \qquad i = 1, 2, \cdots, k \tag{17}$$

where, f_0 is the normal optimum value of the cogging torque obtained by the previous optimization procedure. When the variation of design variable occurs, it is assumed that the system response has distributed variations and the less variance of the system response is more robust. The sequential quadratic programming method has been commonly used to minimize the objective function that satisfies the constraint in this paper [6].

4. Results and Discussion

4.1 Optimal Design Problem of BLDC

In this paper, the analytical model obtained from the RSM is used as objective function or as constraint function. The face center cube is used for the second-order fitted response surface [4, 5].

The levels of three design variables are shown in Table 1. The estimated coefficients of the second-fitted model corresponding to the cogging torque and the running torque are shown in Table 2 and Table 3 respectively.

Table 1 The Level of Design Variables

Design variables	The level of design variables		
	-1 (Low)	0 (Center)	1 (High)
Slot opening: x_1(°)	4	5	6
Stator tooth notching: x_2(°)	4	5	6
Dead zone of magnet pole: x_3(°)	0	3.5	7

Table 2 Estimated Coefficient of the Analytical Model in Cogging Torque

Coefficients	Estimated value	Coefficients	Estimated value
		$\mathbf{b}_{22}$	2.325×10^{-1}
$\mathbf{b}_0$	4.436×10^{-1}	$\mathbf{b}_{33}$	-3.722×10^{-2}
$\mathbf{b}_1$	1.558×10^{-1}	$\mathbf{b}_{12}$	-5.310×10^{-1}
$\mathbf{b}_2$	3.950×10^{-1}	$\mathbf{b}_{13}$	-1.489×10^{-1}
$\mathbf{b}_3$	0.1059×10	$\mathbf{b}_{23}$	-1.578×10^{-1}
$\mathbf{b}_{11}$	2.665×10^{-1}	$\mathbf{b}_{123}$	2.834×10^{-2}

Table 3 Estimated Coefficient of the Analytical Model in Running Torque

Coefficients	Estimated value	Coefficients	Estimated value
		$\mathbf{b}_{22}$	5.467×10^{-2}
$\mathbf{b}_0$	0.9701×10	$\mathbf{b}_{33}$	2.886×10^{-3}
$\mathbf{b}_1$	-0.1370×10	$\mathbf{b}_{12}$	1.278×10^{-1}
$\mathbf{b}_2$	-1.304×10	$\mathbf{b}_{13}$	2.177×10^{-1}
$\mathbf{b}_3$	-1.123×10	$\mathbf{b}_{23}$	1.810×10^{-2}
$\mathbf{b}_{11}$	5.923×10^{-2}	$\mathbf{b}_{123}$	-3.608×10^{-2}

The two analytical models of the objective function and the constraint function are obtained by the observed response corresponding to the peak-to-peak value of the cogging torque and the root mean square value of the running torque respectively. In order to make precise analytical model, the peak-to-peak value of the cogging torque is multiplied by 100 and then root mean square and natural logarithm are taken. Therefore, the objective functions and the constraint function are as follows:

$$\text{Minimize: } f(x) = \hat{y}_{\text{cogging torque}} \quad (N \cdot m) \tag{18}$$

$$\text{Subject to: } g(x) = \hat{y}_{\text{running torque}} \ge 2.3 \ (N \cdot m) \tag{19}$$

$$4 \le x_1 \le 6,\ 4 \le x_2 \le 6,\ 0 \le x_3 \le 7 \tag{20}$$

4.2 Result of the Robust Optimal Design

The sensitivity analysis for each design variable can be done by (15). The aim of the sensitivity analysis is to reduce deviation in the cogging torque by reducing the sensitivity of design variables while maintaining the result of the normal optimization.

Table 4 shows the robust optimization result compare with the normal optimization result. The performance value of the robust optimization result is as good as that of the normal optimization result. The analysis of variance is carried out to investigate improved robustness in the cogging torque. It is assumed that the variations of design variables are in 1(%), 5(%), 10(%) cases, the analyses of variance are archived by using full factorial design concerning to three design variables. In the each case, the variance of the cogging torque is shown in the Table 5. It is shown that the variance of the robust design point is smaller than that of the normal design point at the whole result.

In the similar way, the variance of the running torque in relation to the variations of design variables can be yielded as shown in Table 4. The variance of the robust design point is larger than that of the normal design point at the overall result, but the robust optimal point satisfies the constraints as well as the normal optimal point. The response surfaces of the cogging and the running torque obtained from the RSM are shown in Fig. 3 and Fig. 4 respectively.

Table 4 The Result of the Optimization

Design variables and torques	Normal optimum design	Robust optimum design
Slot opening: x_1(°)	4.85	4.66
Stator tooth notching: x_2(°)	4.79	4.58
Dead zone of magnet pole: x_3(°)	7	7
The Peak-to-peak value of the cogging torque (N·m)	0.011	0.0113
The root mean square value of the running torque (N·m)	2.313	2.319

Table 5 The Variance of the Cogging Torque

The variation of design variables	The variance of the cogging torque	
	Normal optimum design	Robust optimum design
1 (%)	0.119×10^{-5}	0.116×10^{-5}
5 (%)	0.264×10^{-4}	0.245×10^{-4}
10 (%)	0.123×10^{-3}	0.107×10^{-3}

Table 6 The Variance of the Running Torque

The variation of design variables	The variance of the running torque	
	Normal optimum design	Robust optimum design
1 (%)	0.488×10^{-4}	0.557×10^{-4}
5 (%)	0.192×10^{-2}	0.261×10^{-2}
10 (%)	0.157×10^{-1}	0.207×10^{-1}

Fig. 3 The response surface of the cogging torque.

Fig. 4 The response surface of the running torque.

5. Conclusions

This paper presents a robust optimization technique in order to reduce the cogging torque of the BLDC motor. The robust design is accomplish by the minimization of the total sensitivity concerning to three design variables. By the way of the analysis of the variance, It is confirmed that the robust solution less sensitivity than the normal solution. The RSM enables the objective function and the constraint function to be easily created and a great deal of the time in computation to be saved. Therefore, it is expected that the proposed robust design can be easily utilized to enhance the product robustness.

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