Robust Shape Optimization of Electromechanical Devices

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Abstract— **This paper presents the robust shape optimization of electromechanical devices considering the uncertainties of design variables based on numerical optimization technique and finite element method (FEM). In the formulation of robust optimization, the multiobjective function is composed of the mean and the standard deviation of original objective function, while the constraints are supplemented by adding penalty term to original constraints. The sequential quadratic programming (SQP) is applied to solve the robust optimization problem. The results of robust shape optimization considering manufacturing errors are compared with those of conventional shape optimization.**

Index terms—**robust shape optimization, FEM, multiobjective function, mean, standard deviation, manufacturing errors**

I. INTRODUCTION

Many scientists and researchers have developed the characteristics of electromechanical devices using numerical optimization techniques and analysis tools [1]-[3].

Since the deterministic approaches of optimization neglect the effects on variation of design variable such as tolerance, the systems using the conventional deterministic approaches of optimization can not display theirs expected ability or may have the drastic change of performances [4]-[9].

The design of electromechanical devices requires allowance for dimensional and manufacturing tolerances on every part; for example, tolerances occur on stator and rotor punchings, frame dimensions, bearing clearances, magnetic and electric material properties, etc.. The larger the tolerances in the manufacturing process, the lower the cost of manufacturing the machine. Dimensional tolerances, however, can, and often do, impact machine performance, for example, as in operating efficiency and reliability. Therefore, the robust optimal design method is to be inevitably needed considering the uncertainty of design variable in electromechanical devices.

Under the assumption that the design variables are distributed in probability with the tolerance band, the robust optimal design is the technique, which makes the response of system due to the variation of design variables be insensitive so as to approach to the original objective function. The robust optimal design have been actively studied according to the formulation method of the objective function and constraint function and the usefulness has been certified in the many application areas [4]-[9].

In this paper, we introduce the robust shape optimization of electromechanical devices with numerical optimization technique and FEM. In the formulation of robust optimization, the multiobjective function is composed of the mean and standard deviation of original objective function, while the constraints are supplemented by adding penalty term to original constraints [4]. SQP is applied to calculate the design variables [10]-[11]. The results of robust shape optimization, which consider manufacturing errors, are studied and compared with those of conventional shape optimization.

II. PROCEDURE OF ROBUST OPTIMIZATION

General formulation of conventional optimization is expressed as eq. (1).

$$\begin{array}{ll} \textit{Minimize} & f(x) \\ \textit{Subject to} & g_j(x) \le 0, \quad j=1, \cdots, m \\ & x_L \le x \le x_U \end{array} \tag{1}$$

where, $f(x)$ is the objective function, $g_j(x)$ is the constraint function, m is the total number of constraints, and x the vector of design variable and x_L, x_U are the lower bound and upper bound on design variable respectively. Optimal solution derived from eq. (1) is obtained by only the nominal value without considering the uncertainty of design variable. Actually, since the design parameter has the tolerance band, the variation of objective function of eq. (1) occurs. The large variation of objective function causes the performance of system to be unstable. Also, the design based on the optimal solution from eq. (1) may lead to the violation of constraint, it can be useless.

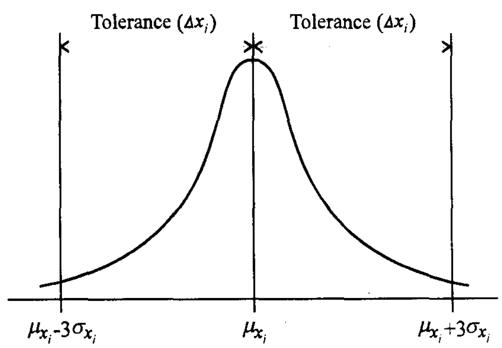

Fig. 1. Mean (μ_{xi}), standard deviation (σ_{xi}) and tolerance band (Δx_i) of the $i-$th design variable.

Manuscript received June 1, 1998.
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Therefore, it is desired to calculate the robust optimal solution, which can decrease the fluctuation of objective function due to the variation of design variable and always satisfy the initial constraints. The variation and distribution of the $i-$th design variable (x_i) is shown in Fig. 1 [5]. In general, the $i-$th design variable (x_i) is regarded as $3\sigma_{xi} = \Delta x_i$ and it is assumed to be distributed between $\mu_{xi} - 3\sigma_{xi}$ and $\mu_{xi} + 3\sigma_{xi}$ [4]-[5].

A. Robustness of Objective Function

Figure 2 describes the robustness of objective function. Point A is the local minimum solution without considering robustness and point B is the optimal solution considering robustness. The performance value of point A is smaller than point B, but the one is very sensitive to the variation of design variable. On the other hand, in case of point B, the performance value is robust, but it is larger one than point A. That is, with the respect of the robust design, point B can be regarded as superior solution to point A. In order to obtain the robust optimal solution, the original objective function can be replaced to the following described multiobjective function ($\Phi(x)$) [4].

$$\begin{aligned} Minimize \quad & \Phi(x) = \alpha \cdot \mu_f + (1-\alpha)\cdot \sigma_f \\ & 0 \le \alpha \le 1 \\ & x_L \le x \pm \Delta x \le x_U \end{aligned} \qquad (2)$$

where, α is the weighting factor determined by the designer and μ_f, σ_f are the mean and standard deviation of objective function respectively.

As shown in Fig. 2, The multiobjective function of eq. (2) tends to have the value of point A when weighting factor equals to 1, while the multiobjective function of eq. (2) tends to have the value of point B when weighting factor equals to zero. The precision value of the mean and standard deviation of eq. (2) can be obtained by using probability density function. However, probability density function is usually unknown or difficult to acquire in electromagnetic field. Even if such information is known, use of probability density function would be computationally time consuming involving finite element analysis and iterative analytical methods. In these problems, first order Taylor expansion is used [9]. The mean and standard deviation of objective function can be approximated as follows using Taylor expansion and neglecting the above second order terms [9].

$$\mu_f = f(x) \qquad (3)$$

$$\sigma_f = \sqrt{\sum_{i=1}^{n}\left(\frac{\partial f}{\partial x_i}\right)_{\bar{x}}^{2} \cdot \sigma_{x_i}^{2}} \qquad (4)$$

where, n is the total number of design variables, $\bar{x}$ is the mean vector of design variable and $\sigma_{x_i}^{2}$ is the pre-known value of design variable instead of probability density function.

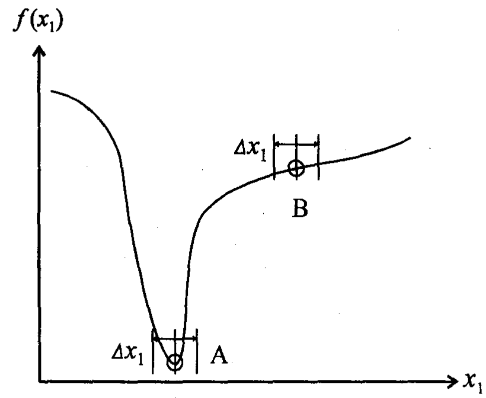

Fig. 2. Robustness of the objective function.

B. Robustness of Constraint Function

If the design variable is distributed, the original constraint function is violated. Figure 3 depicts the robustness of objective function. In Fig. 3, the region, which is represented by dotted line and oblique line, is feasible region without considering the variation of constraint function and the oblique lined region is infeasible region, which is caused by the variation of design variables. If point A is the optimal solution considering the robustness of objective function, point B is the optimal solution under the changed feasible region. In order to consider the variation of constraint, the constraint function can be replaced by eq. (5) [4].

$$g_{newj} = g_j + \sum_{i=1}^{n}\left|\frac{\partial g_j}{\partial x_i}\right|\Delta x_i \qquad (5)$$

where, $\partial g_j / \partial x_i$ is the gradient of the $j-$th constraint function to the $i-$th design variable.

In eq. (5), the newly constructed constraint function (g_{newj}) is adopted to get the conservative value adding the original constraint function (g_j) to the gradient of the one. The penalty term, the second term of right hand in eq. (5), indicates the absolute value regardless of the sign of the gradient.

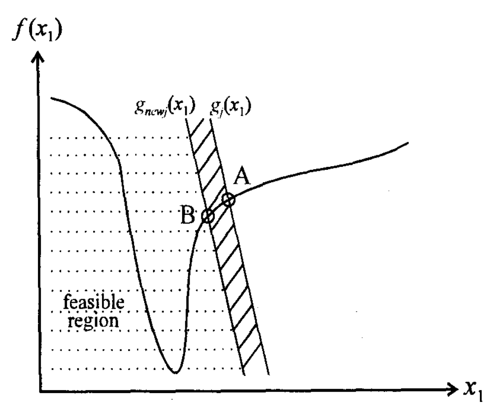

Fig. 3. Robustness of the constraint function.

III. Robust Shape Optimization Problem

SQP technique is applied to find the optimal design parameter [10]-[11]. The SQP converts original objective function and constraint function into linearized subproblem by Taylor expansion. It uses descent condition as a convergence algorithm.

The geometry to be optimized is the pole face of a motor, as shown in Fig. 4, for which the magnetostatic field analysis is carried out in 2-D using FEM. In the model, the stator and the pole face are treated as magnetic material with permeability μ = 3000 (Vs/Am). The winding carries a current of density $\boldsymbol{J}$ = 30000 (A/m^2). The nonlinearity of magnetic material is not taken into account when solving the governing equations of the model.

The goal of the optimization is set as to achieve a sinusoidal magnetic flux density distribution along the line A-B of Fig. 4 positioned 0.12 (mm) below the stator. At point A, magnetic flux density is maximum and it is expected to follow a cosine function to become zero at point B which represents a 90° distance from point A.

The design variables A_1 through A_9 determine the shape of the pole face. These are the y-coordinates of 9 points placed on the pole face in the x direction with fixed x-coordinates. They are allowed to move in the y direction between y = 26(mm) and y = 29.8(mm).

Every time a new mesh is generated, interpolation using a piecewise-cubic splines is applied to each node between design variables.

The objective function that is used by the search process as the decision criteria is obtained from the desired field values(cosine function) and the calculated field values at n points along line A-B using the function

$$f(x)=\sum_{k=1}^{n}\left(\mathbf{B}_{\text{desired, k}}-\mathbf{B}_{\text{calculated, k}}\right)^2 \tag{6}$$

where, **B** is the airgap magnetic flux density.

The constraint is selected as follows;

1) The manufacture of shape must be easy ($A_1 \geq A_2 \geq A_3 \geq A_4 \geq A_5 \geq A_6 \geq A_7 \geq A_8 \geq A_9$).

2) The maximum airgap flux density is less than that of the model in Fig. 4 in order to maintain the initial gap ($g_1(x) = \mathbf{B}_{max} \leq 0.113$ (T)).

IV. Results

The obtained results for the optimization problem descried above are represented in Table I and Figs. 5, 6, 7, and 8. When the optimization is carried out, the converged criteria is determined to 0.1 (%) and the search is converged about 19 iterations. The time cost of robust design is estimated about 15 times than that of the conventional optimization design.

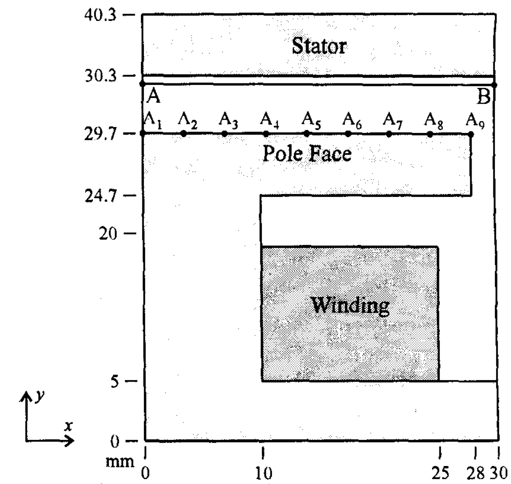

Fig. 4. Geometry of the pole to be optimized and design variables.

The reason is that, in case of the robust design, the objective function and the constraint function are needed to calculate the gradients, which are expressed in eqs. (4) and (5).

Figures 5 and 6 show the optimized pole shape by conventional optimization technique and robust optimization technique along with the geometry of the pole piece and the field lines. The coordinates are summarized in Table I.

Figure 7 depicts the values of mean and standard deviation according to weighting factor. From Fig. 7, it is noted that the values of mean and standard deviation is somewhat linearized when the weighting factor is between 0.5 and 1. But, in case of the weighting factor is below 0.5, it is certified that the values are not linearized.

Finally, in Fig. 8, the airgap flux density along the line A-B according to tolerance is shown. The graph indicates that the larger tolerance band is, the larger the mean and standard deviation of the objective function are. Also, it is noted that it is possible to select the proper tolerance according to the condition of requiring precision so as to reduce the cost of manufacturing the motor.

TABLE I
Coordinates of design variables according to the conventional optimization technique and the robust optimization technique

Design Variable Number	Conventional Optimization Technique (mm)	Robust Optimization Technique (mm)
A_1	(0, 29.6152)	(0, 29.6188)
A_2	(4.6777, 29.590)	(4.6777, 29.5718)
A_3	(8.75, 29.5368)	(8.75, 29.5088)
A_4	(12.8333, 29.4212)	(12.8333, 29.4483)
A_5	(16.3333, 29.2621)	(16.3333, 29.2477)
A_6	(19.8333, 28.9585)	(19.8333, 28.9595)
A_7	(22.75, 28.4492)	(22.75, 28.6061)
A_8	(25.0833, 27.4567)	(25.0833, 27.3151)
A_9	(28, 26.4637)	(28, 26.1763)

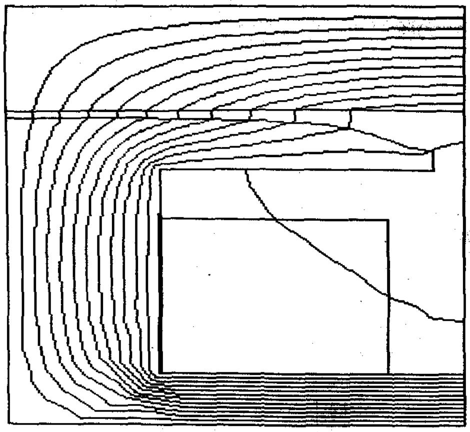

Fig. 5. Optimized pole shape by conventional optimization technique and the field lines.

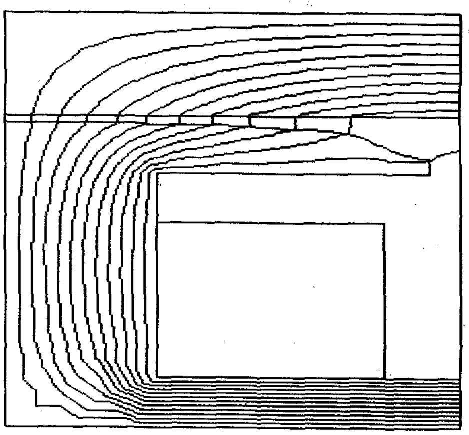

Fig. 6. Optimized pole shape by robust optimization technique and the field lines when the weighting factor (α) is 0.8 and the tolerance (Δx) is 20 (μm).

V. CONCLUSION

In this paper, a robust shape optimization technique of electromechanical devices is presented.

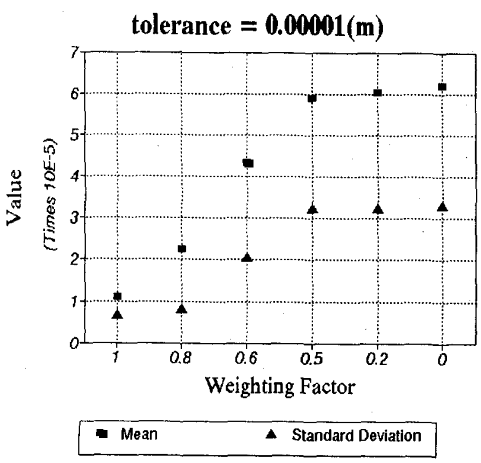

Fig. 7. Values of mean and standard deviation according to weighting factor.

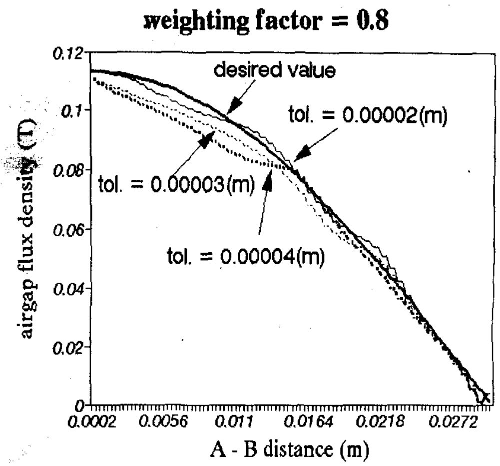

Fig. 8. Airgap flux density along the line A-B according to tolerance.

As a pole shape optimization problem of a electric motor, we can obtain the robust pole face according that the tolerance band is given as input. Also, we make it possible to select the proper tolerance according to the condition of requiring precision so as to reduce the cost of manufacturing the motor.

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