Dynamic Analysis of Radial Force Density in Brushless DC Motor Using 3-D Equivalent Magnetic Circuit Network Method

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Abstract—**The distribution of radial force density in brushless permanent magnet DC motor is not uniform in axial direction. The analysis of radial force density has to consider the 3-D shape of teeth and overhang, because the radial force density causes vibration and acts on the surface of teeth inconstantly. For the analysis, a new 3-D equivalent magnetic circuit network method is used to account the rotor movement without remesh. The radial force density is calculated and analyzed by Maxwell stress tensor and discrete Fourier transform(DFT) respectively. The results of 3-D equivalent magnetic circuit method have been compared with the results of 3-D FEM.**

Index terms—**Brushless DC motor, distribution of radial force density, vibration, remesh, DFT, 3-D EMCN, 3-D FEM.**

I. INTRODUCTION

A brushless permanent magnet DC motor (BLDCM) is popular in a wide variety of position and in a speed control application. However electromagnetic load per volume ratio becomes larger owing to the development of technology and the use of high energy magnet. As the ratio increases, it also gives a bad influence on the vibration and noise in BLDCM.

A principal source of vibration in BLDCM is the induced traveling forces from the rotating permanent magnets acting on the stator. Specially the radial force density acting on the surface of teeth has harmonic components resulting in vibration and noise due to the stator slotting. Therefore, exact analysis of the radial force density is required.

In general, the analysis of force in electromagnetic machines is important for the accurate prediction of motor performance. The force is the source of vibration induced in the devices and cause the serious problem when the forcing frequencies match one or more of the mechanical or structural resonant frequencies in the machine [1], [2].

Many has been studied concerning force calculations by using finite element method(FEM). However, 2-D FEM cannot analyze the radial force density accurately because the radial force density does not act uniformly on the surface of teeth, its distribution is not constant in z direction and the overhang of which length is longer than the height of stator in z direction has to be also considered.

Thus, 3-D FEM is used even though the field distribution calculation time is long because at each different moving step, different meshes are used for fine remesh.

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Also, The use of moving band for the saving the calculation time gives different results changing and twisting the shape of element at each step. In this paper, in order to obtain the accurate radial force density, a new 3-D equivalent magnetic circuit network method (3-D EMCN) is proposed [3]. 3-D EMCN allows modeling of a machine in detail so that the discrete distribution of windings, stator and rotor slotting, and iron saturation can be included. Moreover, the method can cope with the movement simply by applying the distribution of magnetization. It is changed according to relative position of the permanent magnet. 3-D EMCN is simpler in procedure and is shorter in computation time than 3-D FEM, especially when moving of rotor is present. 3-D EMCN uses the initial mesh every each step so that it does not entail remeshing the elements for obtaining the accurate solution. Fixed 9 points on the surface of teeth are selected because the radial force density distribution is time and space variant forcing function induced by the permanent magnet at each point.

The radial force densities are calculated by Maxwell stress tensor and are analyzed by discrete Fourier transform. The results of 3-D EMCN has been compared with the results of 3-D FEM.

II. METHOD OF ANALYSIS

A. 3-D EMCN

Fig. 1 and Fig. 2 show the 3-D analysis model and the cross section of BLDCM respectively. 3-D Equivalent Magnetic Circuit Network (EMCN) of simplified model of BLDCM is also given in Fig. 3. In 3-D EMCN, each region of the machine is divided into elementary volumes (elements) of hexahedral shape. EMCN is built by connecting the centroids(nodes) of adjacent elements with adjacent element's permeance.

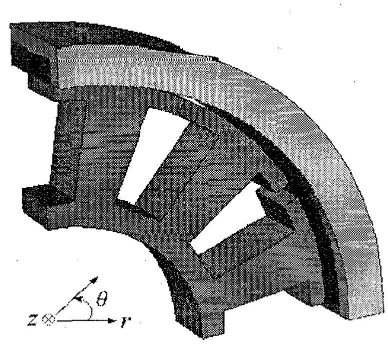

Fig. 1. 3-D analysis 90 degree symmetry model of BLDCM.

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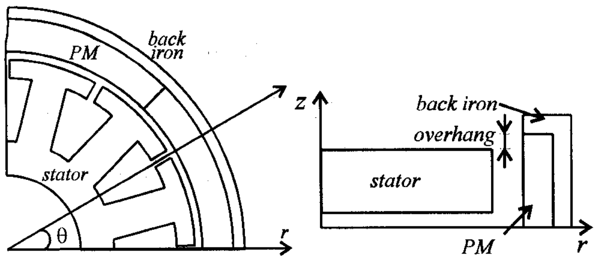

(a) r- θ plane (b) r-z plane

Fig. 2. Cross section of BLDCM.

Flux flow into the node (i,j,k) included in the source region is shown in Fig. 4. The fundamental r-direction node equation between node (i,j,k) and node $(i,j\text{-}1,k)$ can be described by using magnetic scalar potential (MSP) [3].

$$\Phi_{r_{i,j-1,k}} = P_{r_{i,j-1,k}}\left(U_{i,j-1,k} - U_{i,j,k} + E_{i,j-1,k}\right) \tag{1}$$

$$B_{r_{i,j-1,k}} = \Phi_{r_{i,j-1,k}} / S_{r_{i,j-1,k}} \tag{2}$$

$$E_{i,j,k} = NI/m \quad \text{at teeth region} \tag{3}$$

$$= \frac{M\{\theta(i,j,k)\}}{\mu_0 \mu_r} r_{i,j,k} \quad \text{at magnet region} \tag{4}$$

where $\Phi r_{i,j\text{-}1,k}$ is magnetic flux, $Pr_{i,j\text{-}1,k}$ is permeance, $Br_{i,j\text{-}1,k}$ is magnetic flux density, $E_{i,j,k}$ is magnetomotive force of permanent magnet and stator current and $r_{i,j,k}$ is the magnetization depth of permanent magnet between nodes (i,j,k) and $(i,j\text{-}1,k)$. $U_{i,j,k}$ is unknown MSP and $M\{\theta(i,j,k)\}$ is magnetization of permanent magnet at node (i,j,k). N is the number of turns and m is the number of element of the teeth region in the r direction.

Fig. 5 shows the permeance calculation of a cylindrical figured element. Permeance as a function of flux tube geometry is defined by its own area and length as follows:

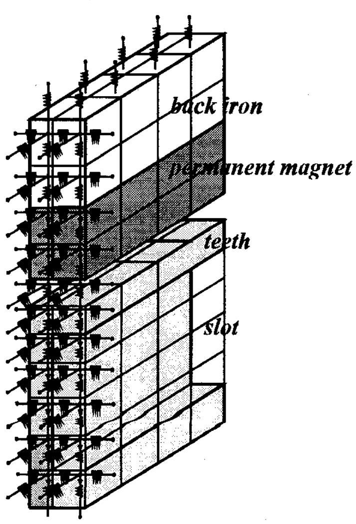

Fig. 3. 3-D equivalent magnetic circuit network of simplified model.

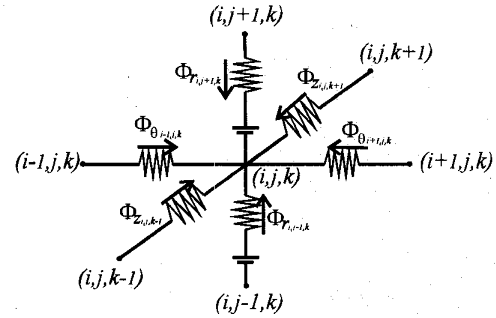

Fig. 4. 3-D equivalent magnetic circuit and flux flow at node (i,j,k).

$$P_r = \mu_0 \mu_r t\theta / ln\left(1+\frac{d}{r}\right) \tag{5}$$

$$P_\theta = \frac{\mu_0 \mu_r t}{\theta} ln\left(1+\frac{d}{r}\right) \tag{6}$$

$$P_z = \frac{\mu_0 \mu_r \theta}{2t}\left(R^2 - r^2\right) \tag{7}$$

where P_r is r direction permeance, P_θ is θ direction permeance and P_z is z direction permeance.

Magnetic flux continuity condition is applied to node (i,j,k) as following :

$$\Phi_{\theta_{i-1,j,k}} + \Phi_{\theta_{i+1,j,k}} + \Phi_{r_{i,j-1,k}} + \Phi_{r_{i,j+1,k}} + \Phi_{z_{i,j,k-1}} + \Phi_{z_{i,j,k+1}} = 0 \tag{8}$$

At node (i,j,k), the node equation is calculated by substituting (1) through (4) into (8). After calculating all nodes by applying the node equation, the system matrix equation is following :

$$[\mathbf{P}]\{\mathbf{U}\} = \{\mathbf{F}\}. \tag{9}$$

where **[P]** is permeance coefficient matrix. It is symmetric and produce good bandwidth and matrix sparsity patterns. **{U}** is MSP matrix of node and **{F}** is forcing matrix (=Permeance × MMF of stator current or PM) Flux density at each node is calculated from system matrix with boundary condition.

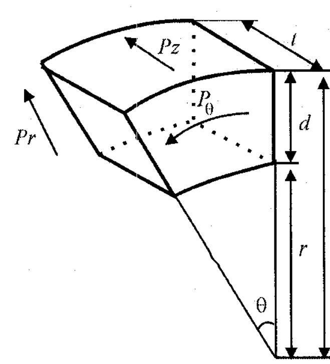

Fig. 5. Permeance calculation for cylindrical figured element with angle θ.

B. Method of Considering the Movement of Rotor

For calculation of time variant magnetic flux it is necessary to analyze dynamic characteristics considering the movement of rotor. In 3-D FEM, calculation of time variant magnetic flux is inconvenient because of time consuming for the remesh at each step. In comparison, 3-D EMCN can solve this problem simply by applying the distribution of magnetization which changes according to relative position of the permanent magnet. That is, as shown in Fig. 6, the variation of magnetization distribution according to the movement of rotor can be represented by 3-D function using the Fourier series such as.

$$M(\theta)=\sum_{n=1}^{\infty}\frac{2pB_r}{n^2\pi^2\theta_d}\left\{sin\frac{\theta_d n\pi}{p}-sin\,n\pi+sin\left(\frac{p-\theta_d}{p}\right)n\pi\right\}\times sin\frac{n\pi}{p}\theta \tag{10}$$

$$M(\theta')=M(\theta-\theta_M) \tag{11}$$

where, p is magnet angle, θ_d is dead zone angle and B_r is residual flux density of permanent magnet. The initial mesh of total element is also used for the analysis without its change according to the movement. Only value of forcing matrix changes in system matrix at each moving step. Therefore, the movement of rotor is easily considered in dynamic characteristic analysis and 3-D EMCN can save computation time remarkably. 3-D FEM takes 4 times cpu time than 3-D EMCN analysis for static characteristic of 90° symmetry model. Some comparisons in computation time, node number and element number are listed in Table I.

The motion of the rotor is governed by the kinetic equation such as,

$$J(dw/dt)+qw=T \tag{12}$$

where J is the rotational inertia, w is the rotor speed, q is the coefficient of viscous friction and T is the electromagnetic torque.

When the rotor rotates, only magnetization distribution $M(\theta)$ changes at each node for $M(\theta')$ according to the rotation angle of the rotor.

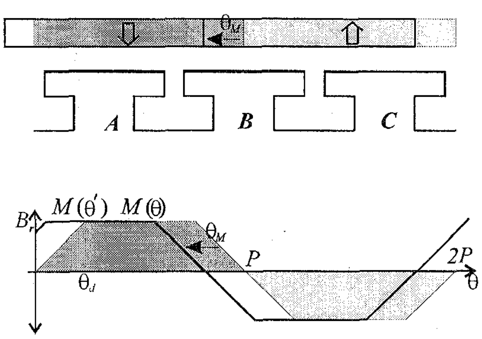

Fig. 6. Variation of magnetization distribution in permanent magnet with rotation angle.

TABLE I.
SOME COMPARISONS OF 3-D FEM AND 3-D EMCN.

Motor Type	3-D FEM	3-D EMCN
8pole/12slot	number of tetrahedral element: 121,728	number of element 29,744
	number of node : 22,509	number of node:29,744
90° symmetry	cpu time: 587 sec	cpu time: 148 sec

Based on the kinetic equation, using the PI speed controller the motor is driven near steady state speed, 5400 rpm which is obtained by giving the 3 phase voltage to the winding of analysis model sequentially at each time step, 0.01ms.

III. ANALYSIS OF RADIAL FORCE DENSITY

A. Force Calculation

The radial force density acting on the surface of teeth are calculated by using Maxwell stress tensor as follows [4]:

$$f_r=\vec{H_1}\left(\vec{B_1}\cdot\vec{n_{12}}\right)-\frac{1}{2}\vec{n_{12}}\left(\vec{B_1}\cdot\vec{H_1}\right)-\left[\vec{H_2}\left(\vec{B_2}\cdot\vec{n_{12}}\right)-\frac{1}{2}\vec{n_{12}}\left(\vec{B_2}\cdot\vec{H_2}\right)\right] \tag{13}$$

where $\vec{H_1}$ and $\vec{H_2}$ is the magnetic field intensity respectively, $\vec{B_1}$ and $\vec{B_2}$ is the flux density of surface element to the boundary respectively, $\vec{n_{12}}$ is unit normal vector in the direction from region 2 to 1.

Frequency analysis of the radial force densities in each point on the surface of the stator is obtained by discrete Fourier transform.

B. Results of 3-D FEM

The exterior-rotor type BLDCM with 8 poles and 12 teeth is used for the analysis. The magnets are radially magnetized.

Fig. 7 presents the mesh of analysis model by 3-D FEM. Because it is difficult for dynamic analysis to consider the movement of rotor, the static characteristic is analyzed by rotating the rotor with 7 degrees. The winding of coil is assumed as a Y winding and the shape of end part in coil is not considered.

Fig. 8 shows the distribution of the flux density vector in the r- θ cross section.

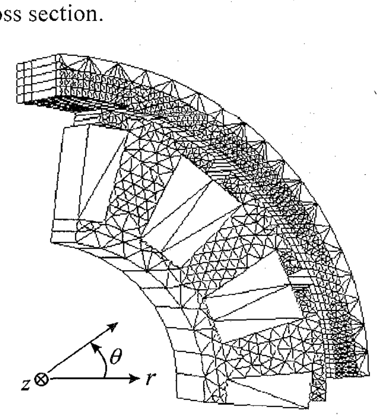

Fig. 7. 3-D FEM mesh of analysis symmetry 90 degree model.

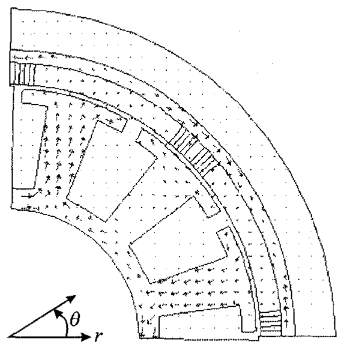

Fig.8. Distribution of flux density vectors in r-z plane by 3-DFEM.

C. Results of 3-D EMCN

Fig. 9 presents the fixed 9 points on the surface of teeth. Fig. 10 and Fig. 11 show time dependent function of radial force density distribution acting at 3 points on the teeth with different integration lines. For a period, force density distributions of the middle point 2 and 5 are relatively symmetric but other points are less symmetric due to flux leakage between magnets and fringing at the stator slot-openings. Fig. 12 shows spectrum analysis at the point 4, 5 and 6 by discrete Fourier transform. The force harmonics was investigated up to 50th. The excitation frequencies are integer multiple of 720 Hz which is the value of multiplication pole number and rotation frequency.

V. Conclusion

In this paper, the radial force densities acting at 9 points on the surface of teeth in BLDCM with overhang, as vibration source, are analyzed by 3-D EMCN considering the movement of rotor. It is simpler procedure, and has relatively short computation time with respect to 3-D FEM because it does not remesh the element at each moving step and the movement of rotor is considered easily in dynamic characteristic analysis. In addition, when it applies to the design for the low noise and vibration in permanent magnet motors such as BLDCM, synchronous motor and stepping motor, we know that it is effective numerical analysis method for the radial force density analysis.

References

[1] B. S. Rahman and D. K. Lieu, "The origin of permanent magnet induced vibration in electric machines," *ASME, Journal of Vibration and Acoustics*, vol. 113, no. 4, pp. 476-481, 1991.

[2] Gunhee Jang, *Analysis of magnetic force and torque in brushless dc motor*, Ph. D dissertation, University of California at Berkeley, USA, 1993.

[3] J. Hur, S. B. Yoon, D. Y. Hwang and D. S. Hyun, "Analysis of PMLSM using 3 dimensional equivalent magnetic circuit network method," *IEEE, Trans. on Magn.*, vol. 33, no. 5, pp. 4143-4145, 1997.

[4] G. Henneberger, Ph. K. Sattler, W. Hadrys, D. Shen, "Procedure for the numerical computation of mechanical vibration in electrical machines." *IEEE, Trans. on Magn.*, vol. 28, no. 2, pp. 1351-1354, 1992.

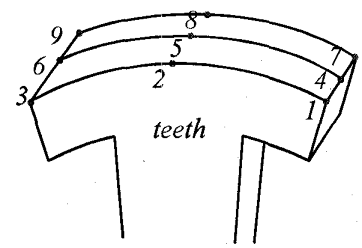

Fig. 9. Fixed 9 points on the surface of teeth.

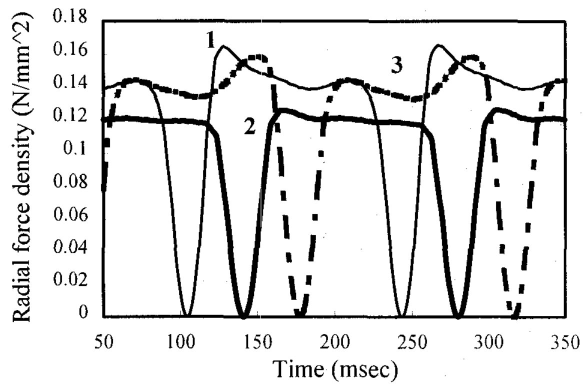

Fig. 10. Radial force density distributions at point 1, 2 and 3.

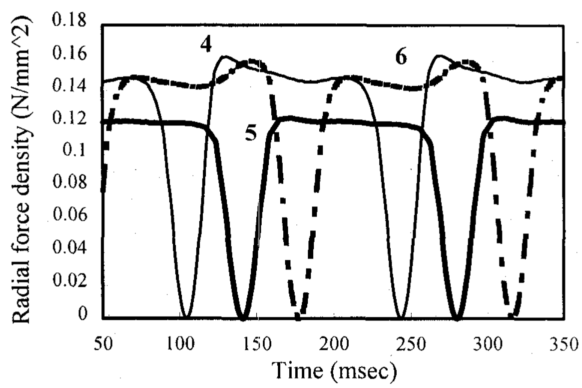

Fig. 11. Radial force density distributions at point 4, 5 and 6.

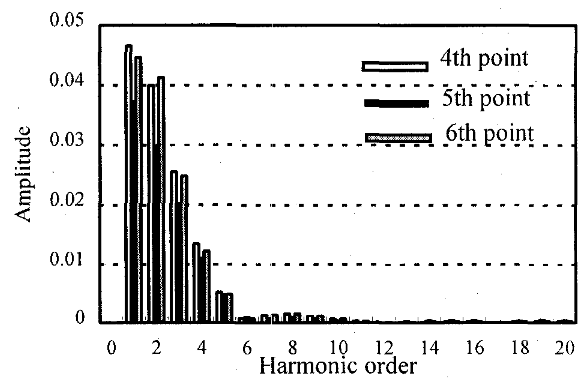

Fig. 12. Spectrum analysis for the point 4, 5 and 6.