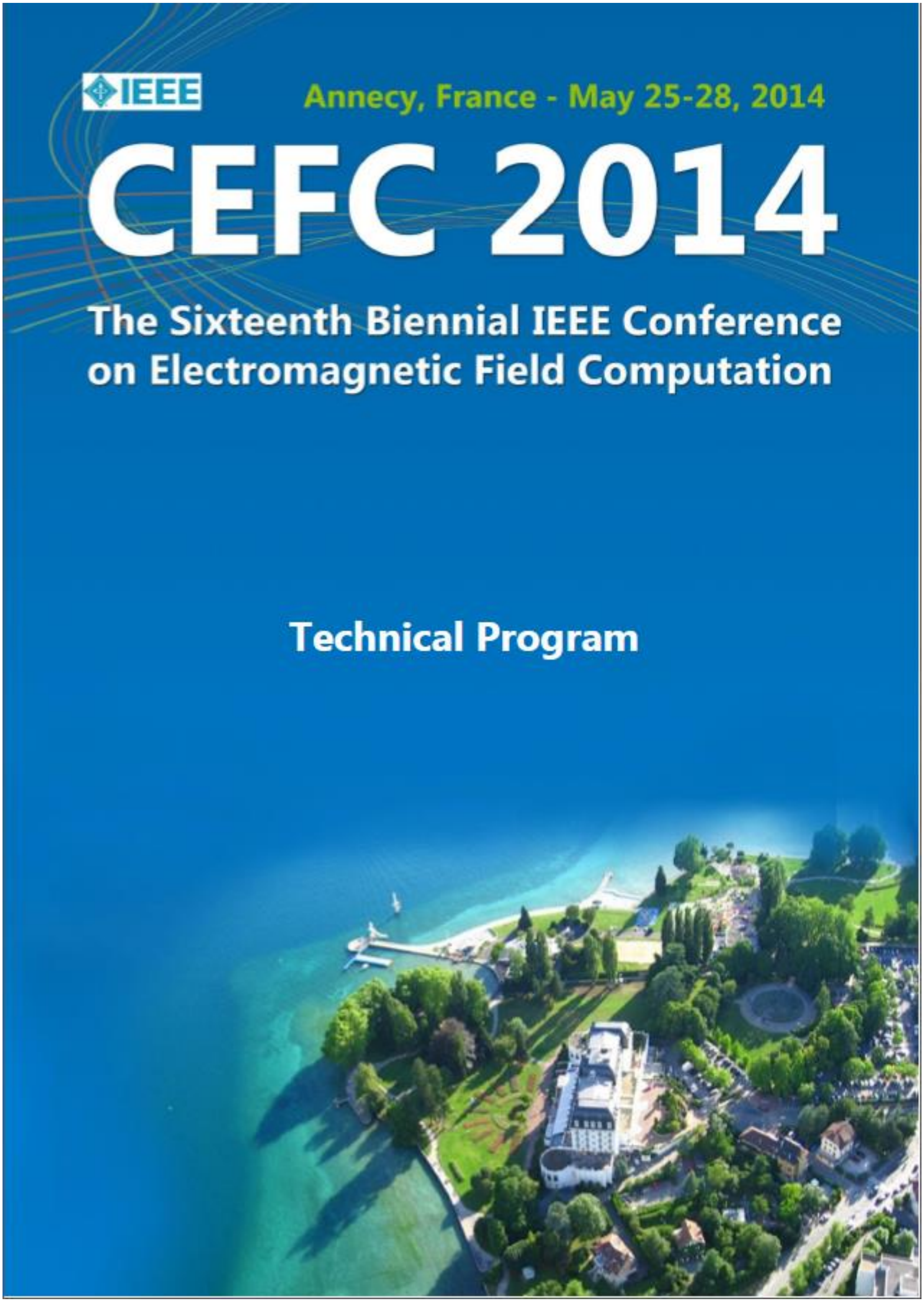
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Research on Helicopter Live-line Work by Platform Method on 1000kV UHV AC Transmission Lines

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The Calculation of Magnetic Field in the Magnetic Core of a two-Winding Transformer under Changing Load

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Scaling Analysis of Radial and Axial Rotary Magnetic Devices

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Calculating Magnetic Fields in Closed Magnetic Cores Using Intero-Differential Equation for the Scalar Potential

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An Analytical Method for Magnetic Field Computation in Surface-Mounted Permanent Magnet Machines with Rotor Eccentricity

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An Improved Tabu Algorithm Applied to Design Optimizations of Electromagnetic Devices

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A simple Procedure to Optimize Small Radius Rounded Corners Obtained from Schwarz-Christoffel Conformal Transformations

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An Analytical Method for Magnetic Field Computation in Surface-Mounted Permanent Magnet Synchronous Machines with Rotor Eccentricity

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An analytical method for calculating the magnetic field in surface-mounted permanent synchronous magnet machines (SPMSM) with rotor eccentricity is suggested. In case of rotor eccentricity, boundary conditions between core and permanent magnet, air and permanent magnet, air and core are not appropriate for cylindrical coordinates due to different thickness of permanent magnet in circumferential direction. In this paper, analytical method of spatial harmonic analysis for SPMSM with rotor eccentricity is suggested. Permanent magnet with eccentricity is divided into pieces and boundary condition is applied separately to each piece and air gap flux density can be obtained by the potential in air gap. Therefore spatial harmonic analysis can be applied with the same governing equations and boundary conditions for motor without eccentricity. The exact solution can be obtained with the suggested method and this method is useful for the analysis of motor with rotor eccentricity.

***Index Terms*—Air gap, Boundary condition, Eccentricity, Magnetic flux density, Magnetic potential, Permanent magnet machine**

I. Introduction

THE instantaneous torque developed by permanent magnet machines and consequently the average and pulsating torques depend on the air gap flux density waveform. This waveform is affected by magnet configuration, air gap length, number of poles, and the direction of magnetization of the magnet [1]. These values are essential for accurate air gap flux density calculation. Analytical method and finite element method are applied to analyze electric motor. Air gap flux density can be easily obtained by spatial harmonic analysis which is one of analytical methods because shape of motor is simplified. Therefore, this method is useful to analyze motor with complex shape. Applying spatial harmonic analysis to permanent magnet machines has been studied for a long time [2]-[4]. Shape of motor is simplified and governing equation is derived in terms of cylindrical coordinates to obtain air gap flux density using spatial harmonic analysis. The potential in air gap and permanent magnet can be obtained by applying boundary conditions of magnetic vector potential or magnetic scalar potential and air gap flux density can be calculated [5]. Characteristics of motor such as back electromotive and cogging torque can be obtained by the air gap flux density. In case of rotor eccentricity, however, boundary conditions between core and permanent magnet, air and permanent magnet, air and core are not appropriate for cylindrical coordinates due to different thickness of permanent magnet in circumferential direction. Therefore, spatial harmonic analysis for permanent magnet machine is approximately conducted due to mathematically difficult boundary condition in cylindrical coordinates.

In this paper, analytical method of spatial harmonic analysis for SPMSM with rotor eccentricity is suggested. Permanent magnet with eccentricity is divided into pieces and boundary condition is applied separately to each piece and air gap flux density can be obtained by the potential in air gap. Therefore, exact solution can be obtained, not the approximate solution. Characteristics of SPMSM can be verified by applying the suggested method.

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II. Computation of Magnetization

1) Stepwise Method

Simplified model of rotor eccentricity is shown in Fig.1 and Fig.2. In stepwise method which is suggested in this paper, permanent magnet is divided into pieces as shown in Fig.1 and Fig.2 with red lines. Therefore, different air gap length can be applied to each pieces due to the different thickness of permanent magnet in circumferential direction. Accordingly, the same general equation and boundary conditions for SPMSM without eccentricity can be used with no mathematical error and exact solution can be obtained.

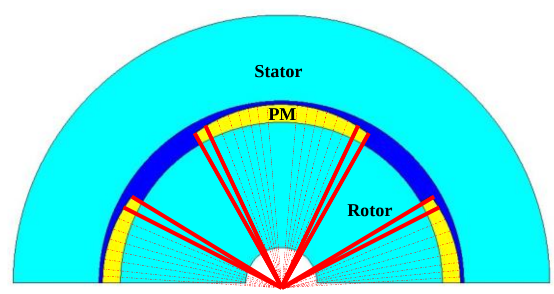

Fig. 1. Shape of rotor eccentricity

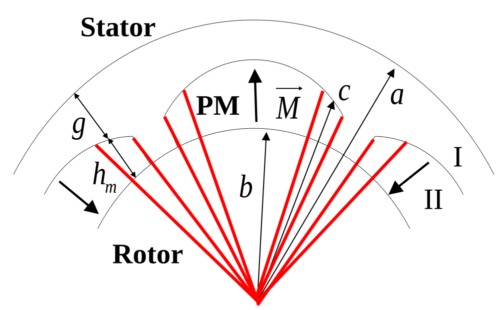

Fig. 2. Geometric configuration of motor for inner rotor type

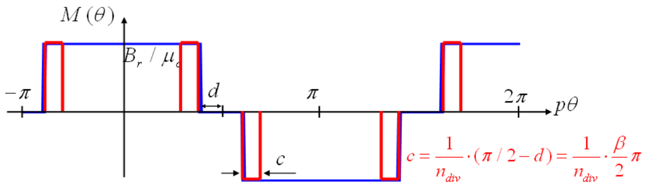

Fig. 3. Stepwise method for calculate waveform of Radial Magnetization

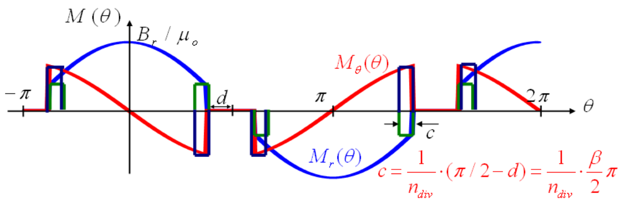

Fig. 4. Stepwise method for calculate waveform of Parallel Magnetization

Geometry of inner rotor type motor is shown in Fig. 2, where a is radius of iron core surface (outer core), b is radius of iron core surface (inner core), c is radius of permanent magnet surface, g is mechanical clearance and h_m is magnetization length.

General equations in terms of polar coordinates are derived for slotless and SPMSM. The field vector $\vec{B}$ and $\vec{H}$ are expressed by

$$\vec{B}_I = \mu_0 \vec{H}_I \tag{1}$$

$$\vec{B}_{II} = \mu_0 \mu_{rec} \vec{H}_{II} + \mu_0 \vec{M} \tag{2}$$

where μ_0 is the permeability of free space, μ_{rec} is the relative recoil permeability, (1) is in the air space, (2) is in the permanent magnet and $\vec{M}$ is the magnetization vector. The amplitude of magnetization $\vec{M}$ is

$$M = \frac{B_r}{\mu_0} \tag{3}$$

where B_r is residual induction.

The magnetization vector in polar coordination system can be expressed by

$$\vec{M} = M_r \vec{a}_r + M_\theta \vec{a}_\theta \tag{4}$$

The analysis works for both a radial and a parallel magnetized rotor.

2) *Radial magnetization*

Permanent magnet is divided into pieces in circumferential direction and the amplitude of magnetization vector is identical for each piece, however, the air gap length varies. Radial magnetization and magnetization of each piece with red lines are shown in Fig. 3. Magnetization for each piece can be expressed as follows

$$M(\theta) = \sum_{k=1,2,3\ldots} \sum_{n=1,3,5\ldots} M_{rn}^k \cos(np\theta) \tag{5}$$

$$M_{rn}^k = \frac{4}{T} \int_{-T/4}^{T/4} M(\theta) \cdot \cos(np\theta) d p\theta \tag{6}$$

$$M(\theta) \ : \ \begin{cases} -\dfrac{\beta\pi}{2}\left(1-\dfrac{k-1}{n_{div}}\right) \le p\theta \le -\dfrac{\beta\pi}{2}\left(1-\dfrac{k}{n_{div}}\right) \\ \dfrac{\beta\pi}{2}\left(1-\dfrac{k}{n_{div}}\right) \le p\theta \le \dfrac{\beta\pi}{2}\left(1-\dfrac{k-1}{n_{div}}\right) \end{cases} \tag{7}$$

$$M_{rn}^k = 2\frac{B_r}{\mu_0}\beta \frac{\sin \dfrac{n\pi\beta}{2}\left(1-\dfrac{k-1}{n_{div}}\right) - \sin \dfrac{n\pi\beta}{2}\left(1-\dfrac{k}{n_{div}}\right)}{\dfrac{n\pi\beta}{2}} \tag{8}$$

$$M_{r\theta}^k = 0$$

where p is pole pair and n_{dv} is divided number

3) *Parallel magnetization*

The same process is applied to parallel magnetization and magnetization for each piece can be expressed as

$$\begin{aligned} \vec{M}(\theta) &= M_r(\theta)\vec{a}_r + M_\theta(\theta)\vec{a}_\theta \\ M_r(\theta) &= \sum_{k=1,2,3\ldots} \sum_{n=1,3,5\ldots} M_{rn}^k \cos(np\theta) \\ M_\theta(\theta) &= \sum_{k=1,2,3\ldots} \sum_{n=1,3,5\ldots} M_{\theta n}^k \sin(np\theta) \end{aligned} \tag{9}$$

$$\begin{aligned} M_r^k(\theta) &= \begin{cases} -\dfrac{\beta\pi}{2}\left(1-\dfrac{k-1}{n_{div}}\right) \le p\theta \le -\dfrac{\beta\pi}{2}\left(1-\dfrac{k}{n_{div}}\right) \\ \dfrac{\beta\pi}{2}\left(1-\dfrac{k}{n_{div}}\right) \le p\theta \le \dfrac{\beta\pi}{2}\left(1-\dfrac{k-1}{n_{div}}\right) \end{cases} \Rightarrow \frac{B_r}{\mu_o} \cdot \cos\theta \\ M_\theta^k(\theta) &= \begin{cases} -\dfrac{\beta\pi}{2}\left(1-\dfrac{k-1}{n_{div}}\right) \le p\theta \le -\dfrac{\beta\pi}{2}\left(1-\dfrac{k}{n_{div}}\right) \\ \dfrac{\beta\pi}{2}\left(1-\dfrac{k}{n_{div}}\right) \le p\theta \le \dfrac{\beta\pi}{2}\left(1-\dfrac{k-1}{n_{div}}\right) \end{cases} \Rightarrow -\frac{B_r}{\mu_o} \cdot \sin\theta \end{aligned} \tag{10}$$

where radial component of magnetization is

if $np \ne 1$

$$M_{rn}^k = \frac{B_r}{\mu_o}\beta \cdot \left[\begin{array}{l} \dfrac{\sin\left\{(np+1)\beta \cdot \dfrac{\pi}{2p}\left(1-\dfrac{k-1}{n_{div}}\right)\right\} - \sin\left\{(np+1)\beta \cdot \dfrac{\pi}{2p}\left(1-\dfrac{k}{n_{div}}\right)\right\}}{(np+1)\beta\dfrac{\pi}{2p}} \\ + \dfrac{\sin\left\{(np-1)\beta \cdot \dfrac{\pi}{2p}\left(1-\dfrac{k-1}{n_{div}}\right)\right\} - \sin\left\{(np-1)\beta \cdot \dfrac{\pi}{2p}\left(1-\dfrac{k}{n_{div}}\right)\right\}}{(np-1)\beta\dfrac{\pi}{2p}} \end{array} \right] \tag{11}$$

if $np = 1 : (n = 1,\ p = 1)$

$$M_{rn}^{k} = \frac{B_r}{\mu_o}\beta\left[\frac{\sin\left\{(np+1)\beta\cdot\frac{\pi}{2p}\left(1-\frac{k-1}{n_{div}}\right)\right\} - \sin\left\{(np+1)\beta\cdot\frac{\pi}{2p}\left(1-\frac{k}{n_{div}}\right)\right\}}{(np+1)\beta\frac{\pi}{2p}} + \frac{1}{n_{div}}\right] \tag{12}$$

where tangential component of magnetization is

if $np \neq 1$

$$M_{\theta n}^{k} = \frac{B_r}{\mu_o}\beta\left[\frac{\sin\left\{(np+1)\beta\cdot\frac{\pi}{2p}\cdot\left(1-\frac{k-1}{n_{div}}\right)\right\} - \sin\left\{(np+1)\beta\cdot\left(1-\frac{k}{n_{div}}\right)\right\}}{(np+1)\beta\cdot\frac{\pi}{2p}} - \frac{\sin\left\{(np-1)\beta\cdot\frac{\pi}{2p}\cdot\left(1-\frac{k-1}{n_{div}}\right)\right\} - \sin\left\{(np-1)\beta\cdot\left(1-\frac{k}{n_{div}}\right)\right\}}{(np-1)\beta\cdot\frac{\pi}{2p}}\right] \tag{13}$$

if $np = 1$

$$M_{\theta n}^{k} = \frac{B_r}{\mu_o}\beta\left[\frac{\sin\left\{(np+1)\beta\frac{\pi}{2p}\left(1-\frac{k-1}{n_{div}}\right)\right\} - \sin\left\{(np+1)\beta\frac{\pi}{2p}\left(1-\frac{k}{n_{div}}\right)\right\}}{(np+1)\beta\frac{\pi}{2p}} - \frac{1}{n_{div}}\right] \tag{14}$$

M_{rn} and $M_{\theta n}$ can be summarized by following equation

$$M_{rn}(\theta) = \frac{B_r}{\mu_0}\beta(A_{1n} + A_{2n})$$

$$M_{\theta n}(\theta) = \frac{B_r}{\mu_0}\beta(A_{1n} - A_{2n})$$

$$A_{1n} = \frac{\sin\left\{(np+1)\beta\cdot\frac{\pi}{2p}\left(1-\frac{k-1}{n_{div}}\right)\right\} - \sin\left\{(np+1)\beta\cdot\frac{\pi}{2p}\left(1-\frac{k}{n_{div}}\right)\right\}}{(np+1)\beta\frac{\pi}{2p}}$$

$$\begin{cases} A_{2n} = 1/n_{div} & \text{for } np = 1 \\ A_{2n} = \dfrac{\sin\left\{(np-1)\beta\cdot\frac{\pi}{2p}\left(1-\frac{k-1}{n_{div}}\right)\right\} - \sin\left\{(np-1)\beta\cdot\frac{\pi}{2p}\left(1-\frac{k}{n_{div}}\right)\right\}}{(np-1)\beta\frac{\pi}{2p}} & \text{for } np \neq 1 \end{cases} \tag{15}$$

III. AIR GAP FIELD DISTRIBUTION

1) Assumptions

An analytical solution for air gap field distribution can be obtained by following assumptions: a) PM have a linear demagnetization characteristic and fully magnetized, b) end-effects are neglected, c) the stator and rotor core have infinite permeability

2) Governing equation

The scalar magnetic potential can be expressed as follows

$$\vec{H} = -\nabla\Phi \tag{16}$$

$$\nabla\times\vec{H} = -\nabla\times(\nabla\Phi) = 0 \quad : \quad \text{null identity.}$$

$$\nabla\cdot\vec{B} = 0, \qquad \vec{B} = \mu\vec{H}$$

Scalar magnetic potential distribution in the air gap is governed by Laplace equation and in the permanent magnet region is governed by Poisson equation.

$$\nabla^2\Phi = 0 \tag{17}$$

$$\vec{B} = \mu_m\vec{H} + \mu_o\vec{M} \tag{18}$$

$$\mu_m = \mu_o\mu_{rec}$$

$$\nabla\cdot\left[\mu_m\vec{H} + \mu_o\vec{M}\right] = \nabla\cdot\left[-\mu_m\nabla\Phi + \mu_o\vec{M}\right] = 0$$

$$\therefore\quad \nabla^2\Phi = \frac{1}{\mu_{rec}}\nabla\cdot\vec{M}$$

where (17) is in the air gap region and (18) is in the permanent magnet region

$$\nabla\cdot\vec{M} = \frac{1}{r}\frac{\partial}{\partial r}(rM_r) + \frac{1}{r}\frac{\partial M_\theta}{\partial\theta} = \frac{M_r}{r} + \frac{\partial M_r}{\partial r} + \frac{1}{r}\frac{\partial M_\theta}{\partial\theta} = \sum_{n=1,3,5\ldots}^{\infty}\frac{1}{r}M_n\cos(np\theta) \tag{19}$$

where

$$M_n = M_{rn} + npM_{\theta n}$$

3) General solution

a) air-gap region (I)

$$\Phi_I(r,\theta) = \sum_{n=1,3,5\ldots}^{\infty}(A_{nI}r^{np} + B_{nI}r^{-np})\cos(np\theta)$$

b) PM region (II)

$$\begin{cases} \Phi_{II}(r,\theta) = \sum_{n=1,3,5\ldots}^{\infty}(A_{nII}r^{np} + B_{nII}r^{-np})\cos(np\theta) + \sum_{n=1,3,5\ldots}^{\infty}\dfrac{M_n}{\mu_{rec}\left[1-(np)^2\right]}r\cos(np\theta) & np \neq 1 \\ \Phi_{II}(r,\theta) = (A_{1II}r + B_{1II}r^{-1})\cos(\theta) + \dfrac{M_1}{2\mu_{rec}}r\ln r\cos(\theta) & np = 1 \end{cases}$$

boundary condition:

$$H_{\theta I}(r,\theta)|_{r=a} = 0$$

$$H_{\theta II}(r,\theta)|_{r=b} = 0$$

$$B_{\theta I}(r,\theta)|_{r=c} = B_{\theta II}(r,\theta)|_{r=c}$$

$$H_{\theta I}(r,\theta)|_{r=c} = H_{\theta II}(r,\theta)|_{r=c}$$

where the dimension a, b, c are defined in Fig. 2.

4) Field Distribution

The magnetic field components in the air gap and magnet regions can be deduced from the general solution of Laplace and Poisson equation and the specific boundary conditions. In the air gap region, following equations are obtained.

$$B_{rI}(r,\theta) = \sum_{n=1,3,5...}^{\infty} K_B(n) \cdot f_{Br}(r) \cdot \cos(np\theta)$$

$$B_{\theta I}(r,\theta) = \sum_{n=1,3,5...}^{\infty} K_B(n) \cdot f_{B\theta}(r) \cdot \sin(np\theta)$$

$$for\ np = 1$$

$$K_B(n) = \frac{\mu_o M_n}{2\mu_{rec}} \cdot \left\{ \frac{A_{3n}\left[(c/a)^2 - (b/a)^2\right] + (b/a)^2 \ln(c/b)^2}{\frac{\mu_{rec}+1}{\mu_{rec}}\left[1-(b/a)^2\right] - \frac{\mu_{rec}-1}{\mu_{rec}}\left[(c/a)^2 - (b/c)^2\right]} \right\}$$

$$f_{Br}(r) = 1 + (a/r)^2$$

$$f_{B\theta}(r) = -1 + (a/r)^2$$

$$A_{3n} = \begin{cases} 2\dfrac{M_{r1}}{M_1} - 1 & \text{for parallel magnetization} \\ 1 & \text{for radial magnetization} \end{cases}$$

$$for\ np \neq 1$$

$$K_B(n) = \frac{\mu_o M_n}{\mu_{rec}} \cdot \frac{np}{(np)^2 - 1} \cdot \left\{ \frac{(A_{3n}-1) + 2(b/c)^{np+1} - (A_{3n}+1)(b/c)^{2np}}{\frac{\mu_{rec}+1}{\mu_{rec}}\left[1-(b/a)^{2np}\right] - \frac{\mu_{rec}-1}{\mu_{rec}}\left[(c/a)^{2np} - (b/c)^{2np}\right]} \right\}$$

$$f_{Br}(r) = (r/a)^{np-1} \cdot (c/a)^{np+1} + (c/r)^{np+1}$$

$$f_{B\theta}(r) = -(r/a)^{np-1} \cdot (c/a)^{np+1} + (c/r)^{np+1}$$

$$A_{3n} = \begin{cases} \left(np - \dfrac{1}{np}\right) \cdot \dfrac{M_{rn}}{M_n} + \dfrac{1}{np} & \text{for parallel magnetization} \\ np & \text{for radial magnetization} \end{cases}$$

These equations are applied to inner rotor type motor, however, it can be applied to outer rotor type as a and b are switched.

Air gap flux density distribution in radial direction and tangential direction obtained by stepwise method are compared to result of finite element method for the verification and are shown in Fig. 5 and Fig. 6. The two models are 4 pole model and outer radius of rotor is 50.5mm. Exactly same result between stepwise method and finite element method are obtained.

IV. Conclusion

A new analytical method for calculating the magnetic field in SPMSM which is stepwise method is suggested. This method is mathematically appropriate and the result is exactly same compared to finite element analysis. Therefore it can be useful for analyzing motor with eccentricity, and furthermore any complex shape can be analyzed with exact solution.

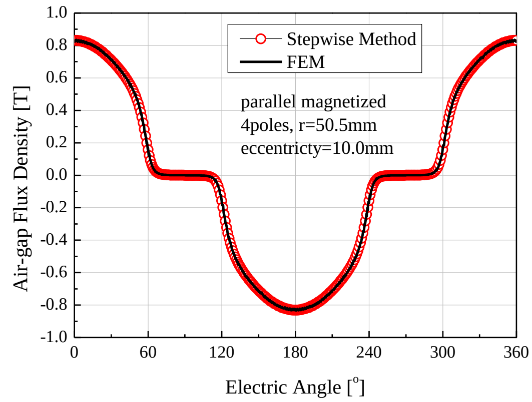

Fig. 5. Comparison of radial flux density between Stepwise Method and FEM with eccentricity

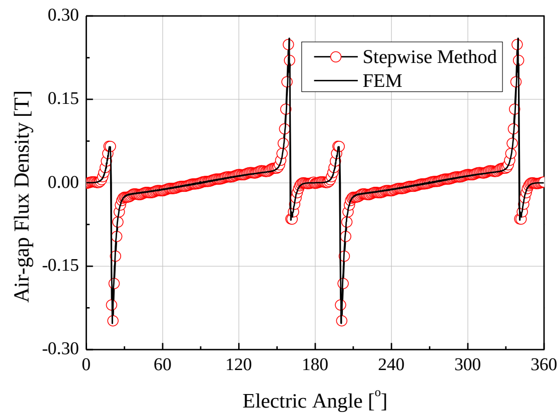

Fig. 6. Comparison of tangential flux density between Stepwise Method and FEM without eccentricity

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