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Design of Linear Oscillating Actuator Using Equivalent Magnetic Circuit Model

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Topics : 10-2

1. Introduction.

In order to achieve the linear reciprocating action, the combination of rotation motor, gear, and crank is used as the conventional solution. There are several inherent problems in this kind of device, such as noise, high mechanical loss, equivalent moving range, respond speed and time delay. By using the linear movement characteristic and MK (mass and spring coefficient) resonance, the linear oscillatory actuator (LOA) can solve all these problems. Therefore, the LOA has been widely applied in the assembly line, home appliances and medical devices. The characteristics of LOA include the displacement and resonance. They are determined by the mechanical parameters. Hence, not like the rotation machine design, the mechanical dynamic should be considered in the design process. The most commonly used method to consider the mechanical dynamic characteristic is to couple the magnetic field, electric circuit and mechanical model in finite element analysis (FEA). This method uses the convergence loop to calculate the current and flux linkage in each given voltage. And then using these current and flux linkage, the force and mechanical equation can be calculated and solved. Although this method can obtain the accurate results, it will consume much computation time and resource. Particularly in the optimization design, each modified LOA model should be solved from the beginning again, which is very non-effective. [1]

In this paper, based on the equivalent magnetic circuit an analytic LOA model is proposed. Using this model the inductance and resistance of LOA can be calculated according to the specification. Using the inductance, a dynamic simulation model of the voltage equation and mechanical equation can be solved. Thus, the current, force, displacement and resonance can be easily solved. This method can not exactly consider the saturation and flux fringing effects, the results hence are not very accurate like the FEA. However, it still can be used to the initial and optimization design because of the similar results fine trend and. Based on a proto coil-type mover LOA, this equivalent magnetic circuit model method will be first introduced in this paper. After obtain the inductance and resistance, the dynamic simulation model will be described and processed. Finally, the analysis results and experiment results will be compared in order to verify the validity of the proposed model and method.

2. Analysis Model

In general, there are three types of LOA. They are permanent magnet mover type, coil mover type and reluctance type. In this paper, a coil mover type LOA will be analyzed. Figure 1 shows the operation principle of the coil type LOA. It can be seen that an AC and a DC voltage sources are applied to the mover coil and stator coil, respectively. Due to the alternative magnetic pole on the teeth of mover, the attractive force and repulsive force are applied to the mover, and hence the mover can do reciprocating move. The dimensions of this coil type LOA is listed in Table 1.

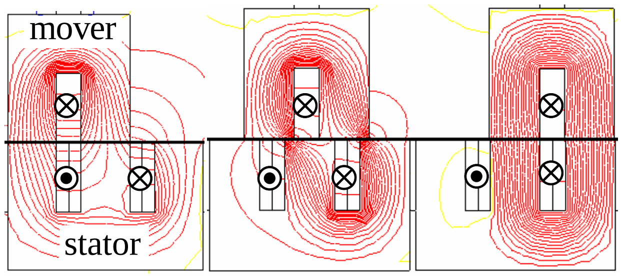


Figure 1: Operation principle of the coil type mover LOA

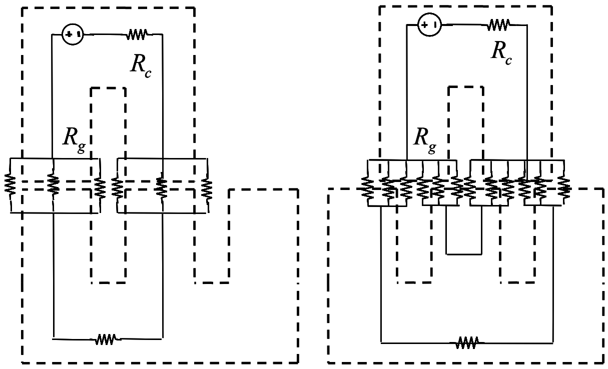
Table 1: Dimensions of analyzed LOA

Item	Value
Stator/ Mover length [mm]	100/62.5
Stator/ Mover height [mm]	65/65
Stator/ Mover tooth width [mm]	25/25
Stack length [mm]	30
No. of stator/mover turns	600/500
Air-gap [mm]	3
Core material	S18

3. Equivalent Magnetic Circuit Model

Equivalent magnetic circuit has been proposed and used in many literatures. [3] By using the ideal of electric circuit, the magnetic flux path can be regarded as a magnetic circuit where the reluctance is corresponding to the electric resistance, the current excited coil, i.e. the magneto-motive force is corresponding to the voltage source, and the flux is corresponding to the electric current. Simplifying the fringing leakage flux and ignoring saturation, the reluctance can be easily calculated according to the dimensions. Due to absence of inductance and capacitance, the magnetic circuit can be solved linearly.

According to the magnetic flux path in the certain mover position, Figure 2 shows the corresponding magnetic circuit models of the analyzed LOA. Figure 2 (a) is the magnetic circuit model in the aligned position, while Figure 2 (b) is the magnetic circuit model in the



(a) (b)

Figure 2: Magnetic circuit model of the analyzed LOA: (a) aligned position; (b) unaligned position.

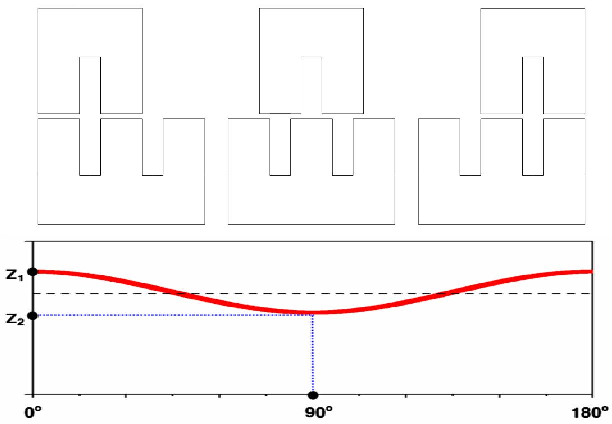


Figure 3: Permeance variation with mover position

unaligned position. As the mover moving, only the reluctances change, these two magnetic circuit models can describe all operation states.

The flux flows through the path which has the lowest reluctance. The reluctance is proportional to the path length and inversely proportional to the area of cross section. As described in (1)

$$R = \frac{l}{\mu A} \tag{1}$$

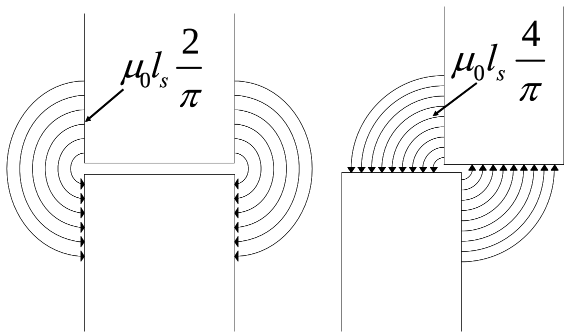
where R is the reluctance, l is the length of flux path, μ is the permeability, and A is the area of cross section of flux path. Figure 3 shows the permeance variation with mover position. It is observed that the permeance can be approximately described in a cosine function of the mover position. Thus, by using this permeance function, the self- and mutual-inductances of the first coil (mover coil) and second coil (stator coil) can be described as (2), (3) and (4),

$$L_1(x) = \frac{1}{4} N_1^{\,2}[(z_1 + z_2) + (z_1 - z_2)\cos(2\pi x/\tau)] \tag{2}$$

$$L_2(x) = N_2^{\,2}[(z_1 + z_2) + (z_1 - z_2)\cos(2\pi x/\tau)] \tag{3}$$

$$L_{12}(x) = N_1 N_2 z_1 \cos(\pi x/\tau) \tag{4}$$

where L_1 is the inductance of the first coil, L_2 is the



(a) (b)

Figure 4: Fringing flux path: (a) fringing flux of aligned position; (b) fringing flux of unaligned position.

inductance of the second coil, L_{12} is the mutual inductance between the first and second coils, N_1 and N_2 are the No. of coil turns of the first coiland second coil, respectively. Additionally, z_1 is the maximum value of the permeance, and z_2 is the minimum value of the permeance. According to Figure 4 (a) and (b), they can be calculated as (5) and (6), respectively. [3]

$$z_1 = \frac{1}{2}\left(2\mu_0 l \frac{2}{\pi} + \mu_0 l \frac{w}{\delta}\right) = \mu_0 l \frac{2}{\pi} + \mu_0 l \frac{w}{2\delta} \tag{5}$$

$$z_2 = 2 \times \frac{1}{2}\left(2\mu_0 l_s \frac{4}{\pi} + \mu_0 l_s \frac{w'}{\delta}\right) = \mu_0 l_s \frac{8}{\pi} + \mu_0 l_s \frac{w'}{\delta} \tag{6}$$

where w is the width of tooth, δ is the air-gap length, and l_s is the stack length. So far, the inductances can be calculated by the dimensions of the LOA. The resistances do not vary with the mover position. They can be simply determined with the coil length, coil area, fill factor, and conductivity.

4. Dynamic Simulation Model

After obtained the inductances and resistances, these inductances and resistances are used to a dynamic simulation model to verify the performance of the LOA. The dynamic simulation model consists of three parts: the electrical dynamic, energy conversion, and mechanical dynamic. [2] The electric dynamic is described by the current differential equations of the stator and mover coils as (7) and (8).

$$e_1 = i_1 R_1 + L_1 \frac{di_1}{dt} + L_{21}\frac{di_2}{dt} + \frac{\partial L_1}{\partial x} i_1 v + \frac{\partial L_{12}}{\partial x} i_2 v \tag{7}$$

$$e_2 = i_2 R_2 + L_2 \frac{di_2}{dt} + L_{12}\frac{di_1}{dt} + \frac{\partial L_2}{\partial x} i_2 v + \frac{\partial L_{21}}{\partial x} i_1 v \tag{8}$$

where v is the velocity of the mover, e_1 is the voltage of the first coil, e_2 is the voltage of the second coil, i_1 is the current of the first coil, and i_2 is the current of the second coil. In theory, due to the DC voltage source, the current differential term of coil 2 should be zero. In practice, however, the variation of the self- and mutual-inductance causes a variation back electromotive force (Back-EMF) in coil 2. Hence there will be a ripple on the total voltage

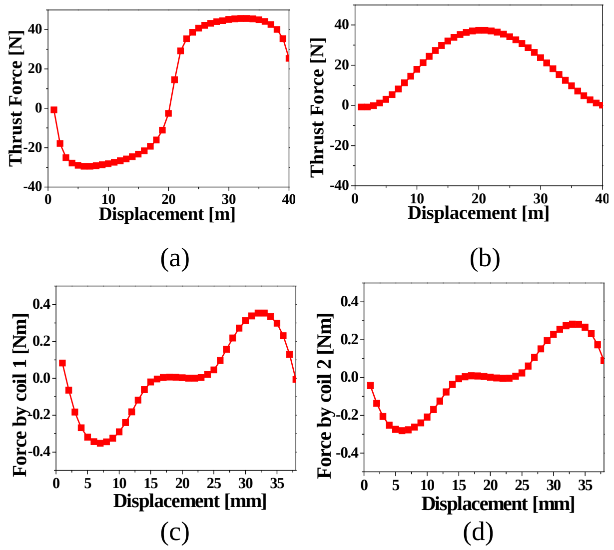


Figure 5: Thrust force: (a) thrust force of proto model; (b) thrust force of optimized model; (c) reduced thrust force of the first coil self-inductance; (d) reduced thrust force of the second coil self-inductance;

of the coil 2, and the current differential term is not zero. The energy conversion, i.e. from the electrical energy to mechanical energy, is presented by (9),

$$F_e = (\frac{1}{2} i_1^2 \frac{\partial L_1}{\partial x} + \frac{1}{2} i_1 i_2 \frac{\partial L_{12}}{\partial x} + \frac{1}{2} i_2^2 \frac{\partial L_2}{\partial x}) \qquad (9)$$

where F_e is the electromagnetic force of the mover. Although the self-inductances of the coil and second coil also contribute the torque in (9), in fact, their mean forces are zero because of their 2-time higher frequency corresponding to the mutual inductance. Figure 5 (a) shows the total thrust force produced by the self- and mutual-inductances. Due to the larger self-inductances, there is inverse direction force between 0 to 20 mm, which much decreases the total thrust force. In order to maximize the force, the variation ratio of the self-inductances of both the first coil and second coil should be reduced. Using a dimension optimization with the proposed magnetic circuit model, the variation ratio of the self-inductances are effectively reduced. The improved force of self-inductance of the first coil and second coil are shown in Figure 5 (c) and (d), respectively. And the minus component of the total thrust force hence is removed as shown in Figure 5 (b).

This obtained force then is used to the mechanical dynamic equation (10) with the mass, spring and damping coefficients.

$$F_e = m\dot{v} + bv + k\int v dt \qquad (10)$$

where m is the mass of mover, b is the damping coefficient, k is the spring coefficient. In order to achieve the MK resonance, the relationship of m, k, and b should be satisfied to (11).

$$f_r = \frac{1}{2\pi}\sqrt{\frac{k}{m} - \frac{b}{4m^2}} \qquad (11)$$

where f_r is the desired resonance frequency. The specification of this LOA system is shown in Table 2. Based on these parameters, the dynamic simulation model is solved by a 4-order Rung-Kutta method.

Table 2: Specification of the analyzed LOA model

Item	Value
Input voltage of 1st coil [V_{peak}]	16.4
Input voltage of 2nd coil [V]	23
Input frequency of 1st coil [Hz]	8
Initial displacement of mover [mm]	18.75

The current in first coil is a sinusoidal waveform, while the current in the second coil is a half waveform with 50Hz frequency. The frequency of the current in the first coil should be lower than current frequency of the second coil. If they are similar, the mover will not reciprocate. Due to the much higher frequency, the current in the second coil can be regarded as DC waveform.

Figure 6 (a) shows displacement of the mover by the dynamic simulation. It is obvious that there is a transient state in the beginning, and the steady state is achieved after 3 second. Figure 6 (b) shows the reluctance force. Due to the small variation ratio of self inductance, the reluctance force is almost 0N. It can be seen in Figure 6 (c), however, that the thrust force which is produced by variation ratio of mutual inductance is quite large. In theory, the oscillator can work with the higher frequency in the second coil. However, the higher frequency implies the special control unit. In the case that the initial mover does not align with the central point of the stator, the mover does not reciprocate continuously. It will just moves to one side. In practice, there must be a spring that is used to decide the initial point.

5. Experiment and Comparison

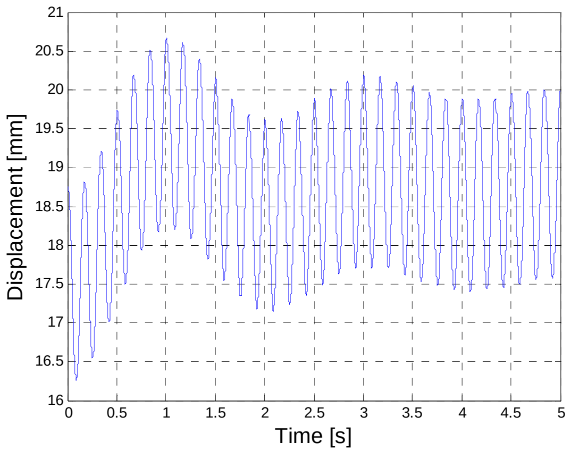
A proto LOA shown in Figure 7 is manufactured and used to verify the validity of the analysis result. The half-wave voltage source of the second coil is generated by a two-diode rectifier circuit. A linear guide is used to support the mover and keep it move linearly. If the voltage frequency in the first coil is very low, the mover can not reciprocate continuously. If the frequency is very high, the measurement for displacement will be difficult because of the short displacement. Table 3 shows the comparison of the analysis results and experiment results. In the different input frequency, 8 Hz and 10 Hz, it can be seen that the displacement and current are similar. Considering the simplicity of this method and the application, the results are acceptable, especially in the initial design step.

6. Conclusion

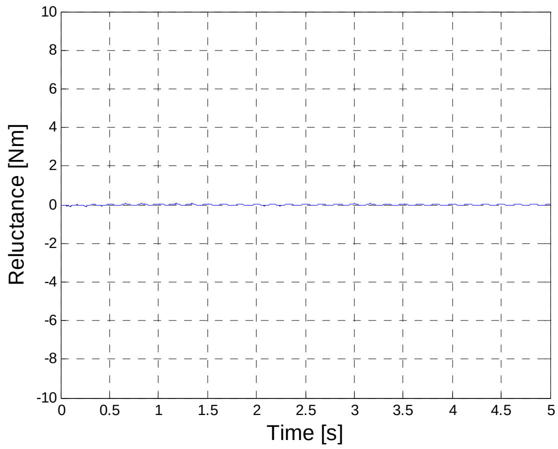
This paper proposed an equivalent magnetic circuit model for the linear oscillatory actuator. Depending on this model, the inductances and resistances of a coil type linear oscillatory actuator are successfully calculated. With assistance of a dynamic simulation model, the performance of the linear oscillatory actuator can be approximately predicted. The analysis results are very similar to those of experiment. This equivalent magnetic circuit model will be very helpful in the linear actuator initial and optimization design.

Table 3: Comparison of analysis and experiment results

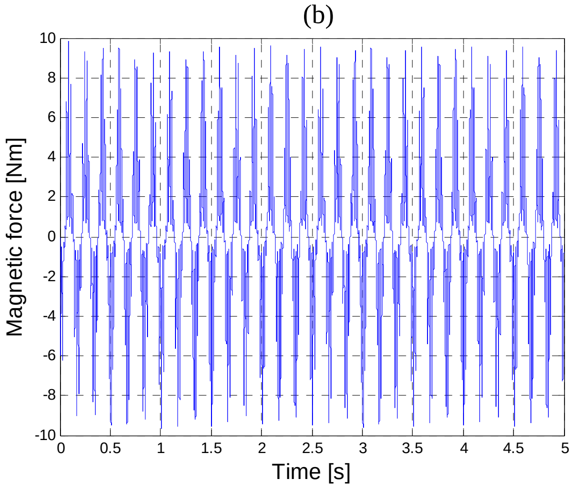
		Comparison items			
		8 Hz		10 Hz	
Voltage [V_{rms}]		Disp.	Cur.	Disp.	Cur.
Experiment	11.6	2	1.43	0.8	1.32
Analysis		1.5	1.77	0.7	1.73



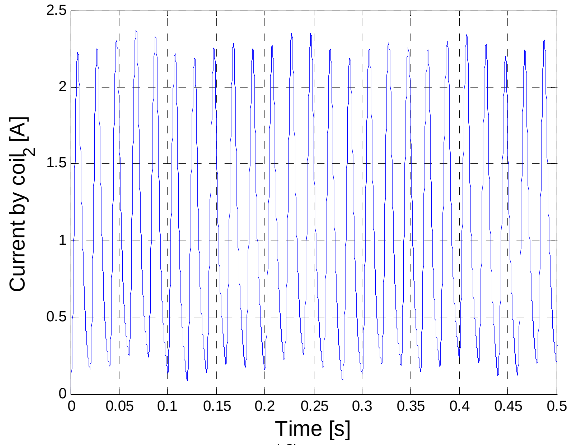
(a)



(b)



(c)



(d)

Figure 6: Simulation results: (a) displacement; (b), reluctance force; (c) thrust force; (d) current in second coil

Figure 7: Proto linear oscillatory actuator

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