

A Torque Ripple Reduction Drive Strategy for Permanent Magnet BLDC Motor with Imperfect Back-EMF

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Abstract—This paper presents a novel drive strategy to reduce torque ripple of Permanent Magnet Brushless DC (BLDC) Motor with imperfect 120° flat top Back Electromotive Force (Back-EMF). In this strategy, the Back-EMF is divided into four sections. Then, in the each section the phase-currents are regulated by corresponding PWM duty-ratio to compensate the torque ripple caused by imperfect Back-EMF. A program based on this strategy has been implemented in MATLAB@Simulink. The validity of the presented method is verified by the results.

Index Terms—BLDC Motor, Drive, PWM, Torque Ripple.

I. INTRODUCTION

In Permanent Magnet Brushless DC (BLDC) Motor drive, the commutation process may produce a current ripple in the non-commutated phase. In ideal Back Electromotive Force (Back-EMF) case, it directly causes a ripple in torque [1]. Many solutions have been introduced to reduce the commutation current ripple such as [2] [3] and [4]. In practice, however, the Back-EMF usually has an imperfect 120° flat top as shown in Fig. 1 (a). According to the conventional solutions, the torque for this imperfect Back-EMF is simulated and shown in Fig. 1 (b). Because the torque ripple achieves about 30% of mean torque value, the conventional solutions are valid no more.

In this paper, a drive strategy of torque ripple reduction for the BLDC Motor with imperfect Back-EMF is presented. In this strategy, the phase-back-EMF of every 60° is divided into four sections: constant Back-EMF section, Back-EMF decreasing section, commutation section, and after commutation section. First, in the constant Back-EMF section, according to torque equation, a set-point constant torque value is determined. And then, in the other sections, the phase-current is regulated by corresponding PWM duty-ratio to compensate the ripple of Back-EMF. In commutation section, the conventional commutation torque ripple estimation equations should be modified. An improved criterion is proposed in this section. Depending on it, the torque ripple states can be predicted easily.

As the above mentioned, the BLDC motor is driven by different PWM duty-ratio according to each Back-EMF section, which needs a simple speed controller. Due to the all drive sections are mainly determined by Back-

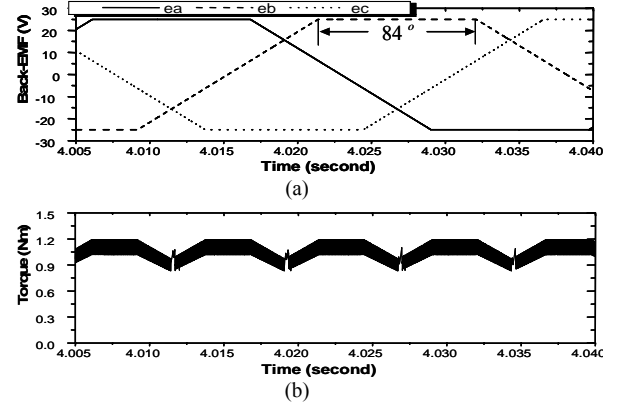


Fig. 1 (a) the imperfect Back-EMF shapes with 84° flat top; (b) the torque ripple using the conventional BLDC motor drive approach.

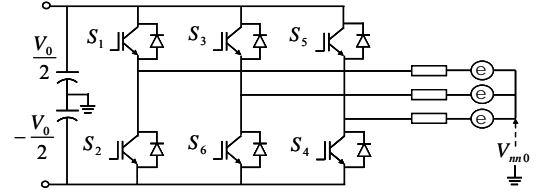


Fig. 2 the equivalent circuit of BLDC motor drive

EMF shape and rotor position, the robust performance is achieved. A program based on this drive strategy has been successfully implemented in MATLAB@Simulink. The validity of the presented method is verified by it.

II. TORQUE RIPPLE ANALYSIS AND PROPOSED STRATEGY

Assume 3-phase windings are “Y” connected, the permanent magnet is surface type, and the motor has a symmetrical structure. The equivalent circuit of a BLDC motor drive is shown in Fig. 2. The motor state equation is expressed as

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \begin{bmatrix} R & 0 & 0 \\ 0 & R & 0 \\ 0 & 0 & R \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \begin{bmatrix} L & 0 & 0 \\ 0 & L & 0 \\ 0 & 0 & L \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \begin{bmatrix} e_a \\ e_b \\ e_c \end{bmatrix} + \begin{bmatrix} V_{n0} \\ V_{n0} \\ V_{n0} \end{bmatrix} \quad (1)$$

where the v_x is each phase voltage, i_x is each phase-current, and e_x is each phase-back-EMF (where the x means a,b or c). In addition, R , L and V_{n0} are phase

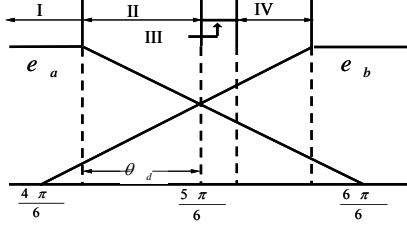


Fig. 3 the chosen part of Back-EMF and divided sections

resistance, phase inductance and neutral voltage, respectively. The electrical-to-mechanical energy convert equation is

$$T = \frac{1}{\omega} (e_a i_a + e_b i_b + e_c i_c) \quad (2)$$

where T and ω denote torque and angular velocity, respectively [1].

Owing to symmetry, an arbitrary section of 3-phase-back-EMFs shown as Fig. 3 is chosen and analyzed. In the Fig. 3, the Back-EMF of phase-c is ignored because of constant value. And the θ_d is the half short angle of Back-EMF flat top.

Where the I is constant Back-EMF section, the II is Back-EMF decreasing section, the III is commutation section, and the IV is after commutation section.

A. Constant Back-EMF Section

During this section, only two phases are excited. In steady state, the set-point current may be determined by (3). And depending on the parameter of Table I, the PWM are set as 10k Hz and 4k Hz, respectively [4].

$$e_a i_a + e_c i_c = 2EI \quad (3)$$

B. Back-EMF Decreasing Section

As Fig. 3, the Back-EMF of phase-a begins varying in this section. It is to lead the torque decreases if the phase-a current is held. In order to compensate the varying of Back-EMF and hence maintain the torque, the current should be regulated properly. According to (3) and Back-EMF equation (4) and (5), the corresponding varying current may be found to be (6).

$$e_a = \frac{E(\pi - \theta)}{\pi/6 + \theta_d} \quad (4)$$

$$e_c = -E \quad (5)$$

$$\frac{di_a}{dt} = \frac{2I\omega(\pi/6 + \theta_d)}{(7\pi/6 + \theta_d - \theta)^2} \quad (6)$$

Assume a complementary switching method is used to inverter operation. Owing to only two excited phases, the motor state equation is simplified as

$$\begin{aligned} \frac{V_0}{2} S_2 &= L \frac{di_a}{dt} + e_a + V_{m0} \\ -\frac{V_0}{2} S_2 &= L \frac{di_c}{dt} + e_c + V_{m0} \end{aligned} \quad (7)$$

The two states of varying current may be expressed as

$$\begin{aligned} \frac{di_a}{dt} &= \frac{V_0}{2L} - E \frac{(23\pi/6) - 4\theta - \theta_d}{3L(\pi/6 + \theta_d)} (S_2 = 1) \\ \frac{di_a}{dt} &= -\frac{V_0}{2L} - E \frac{(23\pi/6) - 4\theta - \theta_d}{3L(\pi/6 + \theta_d)} (S_2 = -1) \end{aligned} \quad (8)$$

By means of the State-Space Average Technique (SAT) [5], the PWM duty-ratio function to maintain current reference varying trend is worked out and expressed as

$$D_2 = \frac{2I\omega L(\pi/6 + \theta_d)}{V_0(7\pi/6 + \theta_d - \theta)^2} + \frac{E(23\pi/6 - 4\theta - \theta_d)}{3V_0(\pi/6 + \theta_d)} + \frac{1}{2} \quad (9)$$

C. Commutation Section

Exist of inductance makes current be not able to vanish or establish to set-point value immediately. Therefore, during the commutation section, there are currents in all 3 phases. Usually, the commutation duration is so short that the Back-EMF in this section could be regarded as constants. The Back-EMF equations are shown in

$$e_a = \frac{\pi E}{\pi + 6\theta_d} \quad (10)$$

$$e_a = \frac{\pi E}{\pi + 6\theta_d} \quad (11)$$

$$e_c = -E \quad (12)$$

The expected phase-a current varying rate is

$$\frac{di_a}{dt} = -\frac{(V_0 + 2E)(\pi + 6\theta_d) + E\pi}{3L(\pi + 6\theta_d)} \quad (13)$$

In addition, if any phase is not operated by switch device during commutation, the natural establishing rate of phase-b may be worked out by (14) and expressed as (15).

$$-\frac{V_0}{2} = L \frac{di_a}{dt} + e_a + V_{m0} \quad (14)$$

$$\begin{aligned} \frac{V_0}{2} &= L \frac{di_b}{dt} + e_b + V_{m0} \\ -\frac{V_0}{2} &= L \frac{di_c}{dt} + e_c + V_{m0} \end{aligned}$$

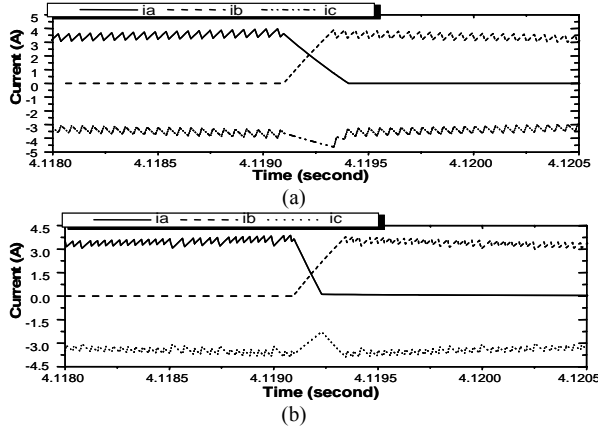
$$\frac{di_b}{dt} = \frac{(V_0 - E)(2\pi + 6\theta_d) + 6\theta_d V_0}{3L(\pi + 6\theta_d)} \quad (15)$$

1) Case I: If minus (13) equals to (15)

$$-\frac{di_a}{dt} = \frac{di_b}{dt} \quad (16)$$

The decreasing torque of phase-a may be counteracted by the increasing of phase-b and non-commutation phase, phase-c, has not current ripple, i.e. the phase-c torque is constant. An estimation equation is gotten by simplifying (16) and shown as

$$\frac{V_0}{E} = \frac{4\pi + 12\theta_d}{\pi + 6\theta_d} \quad (17)$$



2) *Case II:* If minus (13) is smaller than (15)

$$-\frac{di_a}{dt} < \frac{di_b}{dt} \quad (18)$$

This means the decreasing rate of phase-a current is slower than the increasing rate of phase-b current. Before phase-a current vanishes, phase-b current has been set-point value. According to KCL, the phase-c current is increased suddenly as shown in Fig. 4 (a). An estimation equation is gotten by simplifying (18) and shown as

$$\frac{V_0}{E} > \frac{4\pi + 12\theta_d}{\pi + 6\theta_d} \quad (19)$$

In order to slow the increasing rate of phase-b current, it prefers to switch it by PWM. The corresponding voltage equations are given by

$$\begin{aligned} -\frac{V_0}{2} &= L \frac{di_a}{dt} + e_a + V_{m_0} \\ \frac{V_0}{2} S_{32} &= L \frac{di_b}{dt} + e_b + V_{m_0} \\ -\frac{V_0}{2} S_{32} &= L \frac{di_c}{dt} + e_c + V_{m_0} \end{aligned} \quad (20)$$

Also, by means of SAT, the PWM duty-ratio is found to be

$$D_{32} = \frac{2}{3} + \frac{4E(\pi + 3\theta_d)}{3V_0(\pi + 6\theta_d)} \quad (21)$$

3) *Case III:* If minus (13) is greater than (15)

$$-\frac{di_a}{dt} > \frac{di_b}{dt} \quad (22)$$

This means the decreasing rate of phase-a current is faster than the increasing rate of phase-b current. Before phase-b current achieve set-point value, phase-a has vanished. It leads that the phase-c current is reduced as shown in Fig.4 (b). An estimation equation is gotten by simplifying (22) and shown as

$$\frac{V_0}{E} < \frac{4\pi + 12\theta_d}{\pi + 6\theta_d} \quad (23)$$

The solution is the same with case II. Nevertheless, the switching effect to neutral voltage may influence phase-b current when phase-a current is being regulated [5]. The neutral voltage equation in this case is

$$V_{m_0} = -\frac{V_0 S_{33}}{6} - \frac{e_a + e_b + e_c}{3} \quad (24)$$

In different switch state, the current varying rates of phase-a and b are

$$\frac{di_a}{dt} = \frac{-V_0 - 2e_a + e_b + e_c}{3L} (S_{33} = 1) \quad (25)$$

$$\frac{di_a}{dt} = \frac{V_0 - 2e_a + e_b + e_c}{3L} (S_{33} = -1)$$

$$\frac{di_b}{dt} = \frac{2V_0 - 2e_b + e_a + e_c}{3L} (S_{33} = 1) \quad (26)$$

$$\frac{di_b}{dt} = \frac{V_0 - 2e_b + e_a + e_c}{3L} (S_{33} = -1)$$

So, the PWM duty-ratio for case III is

$$D_{33} = 2 - \frac{2E(2\pi + 6\theta_d)}{V_0(\pi + 6\theta_d)} \quad (27)$$

D. After Commutation Section

After commutation, only phase-b and phase-c are excited, and the Back-EMF of phase-b continues increasing. This is the similar condition with Back-EMF Decreasing Section. Hence, in the same method, the PWM duty-ratio is given by

$$D_4 = \frac{2I\omega L(\pi/6 + \theta_d)}{V_0(\theta + \theta_d - \pi/2)^2} + E \frac{2\theta + \theta_d - (7\pi/6)}{3V_0(\pi/6 + \theta_d)} + \frac{1}{2} \quad (28)$$

III. SIMULATION RESULTS VERIFICATION

Based on the proposed strategy, the topology of this BLDC motor simulation program is shown in Fig. 5. Specially, the PWM duty-ratio selector block is shown in Fig. 6. As the Back-EMF shape in Fig. 1 (a) and the

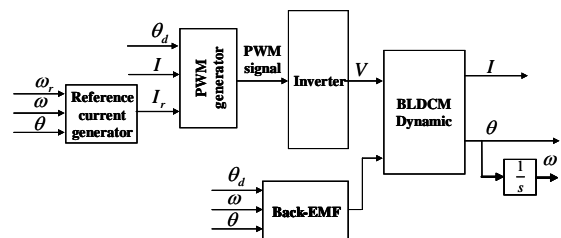


Fig. 5 the topology of simulation

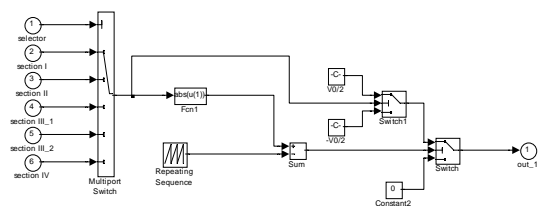


Fig. 6 the PWM duty-ratio selector block

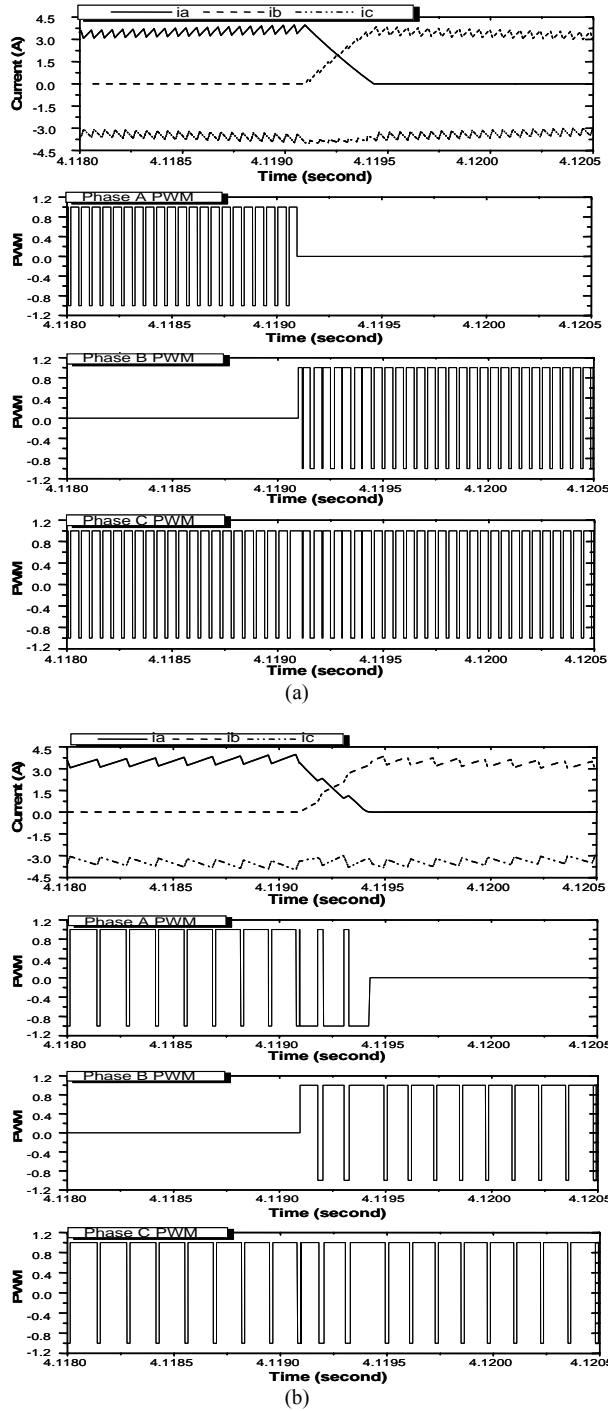


Fig. 7 (a) the simulation results of case II; (b) the simulation results of case III

parameters of Table I, the simulation results are shown in Fig. 7. The difference of Fig. 7 (a) and (b) is commutation section condition. The (a) is simulated for case II, while (b) is for case III.

Fig. 8 (a) and (b) respectively are the current and torque results in case II. As the results showing, the phase current shape is not rectangular which leads that the torque ripple is reduced by about 10% of mean value.

IV. CONCLUSIONS

In BLDC motor, both the imperfect Back-EMF shape

and the commutation current ripple may bring about torque ripple. In order to reduce it, many solutions based on ideal Back-EMF shape have been presented. In practice, however, the imperfect Back-EMF shape makes the conventional solutions valid no more. The torque ripple still achieves about 30% of mean torque value.

In this paper, a drive strategy which focuses on imperfect Back-EMF is proposed. This strategy divides Back-EMF into four sections. And according to different variables of these sections, the corresponding PWM duty-ratio is calculated and to regulate current shape. Owing to the all drive sections are mainly determined by Back-EMF shape and rotor position, the robust performance is achieved. By means of this strategy, the torque ripple may be reduced to 10% of mean torque value.

TABLE I
SPECIFICATIONS OF BLDC MOTOR

No.	Parameters	Value	Unit
1	Phase Inductance	3.05	mH
2	Phase Resistance	0.75	Ohm
3	Rated Speed	1300	RPM
4	DC-Link Voltage in case II	80	V
5	DC-Link Voltage in case III	110	V
6	Back-EMF of case II (in rated speed)	30	V
7	Back-EMF of case III (in rated speed)	25	V

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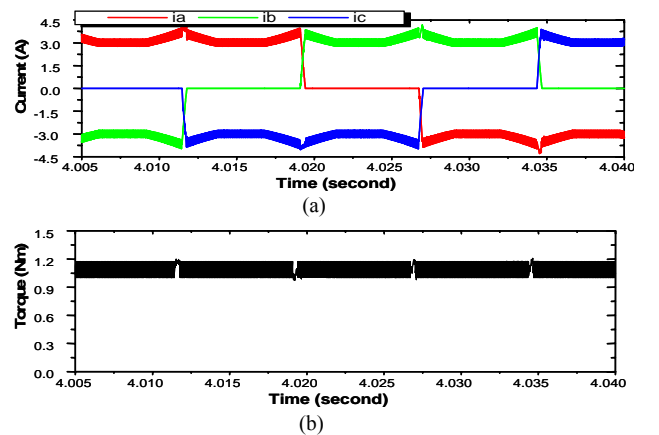


Fig.8 (a) the current shape of proposed strategy; (b) the improved torque



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Sincerely,

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Session DS3F1

PM Machines and Drives (5)

Date: Wednesday, 22 November 2006

Time: 14:00-15:20

Venue: Room F

DS3F1-01 PDF	Magnetic Field Analysis of Permanent Magnet Synchronous Motor Using the Transfer Relations Seok-Myeong Jang, Kyoung-Jin Ko, Han-Wook Cho, Jang-Young Choi Chungnam National University, Korea
DS3F1-02 PDF	A Torque Ripple Reduction Drive Strategy for Permanent Magnet BLDC Motor with Imperfect Back-EMF Tao Sun ¹⁾ , Geun-Ho Lee ²⁾ , Jung-Pyo Hong ¹⁾ ¹⁾ Changwon National University, Korea, ²⁾ Namhae College, Korea
DS3F1-03 PDF	The Design and Analysis of a High Efficiency Permanent Magnet Reluctance Motor Peng Zhang, Soon-O Kwon, Jung-Pyo Hong Changwon National University, Korea
DS3F1-04 PDF	Comparison of Motor Parameters and Output Characteristics of IPMSMs with Concentrated and Distributed Windings Soon-O Kwon, Su-Beom Park, Zhang Peng, Liang Fang, Jung-Pyo Hong Changwon National University, Korea
DS3F1-05 PDF	Robust Flux-Weakening Control of Permanent Magnet Synchronous Machines Incorporating Speed Regulation Song Chi ¹⁾ , Longya Xu ¹⁾ , Jinsheng Sun ²⁾ ¹⁾ The Ohio State University, USA, ²⁾ Hebei Polytechnic University, China
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DS3F1-08 PDF	Study on the Static Characteristics of a Hybrid Stepping Motor by Combining Magnetic Circuit Method and Numerical Magnetic Field Analysis Yiping Dou ¹⁾ , Youguang Guo ²⁾ , Jianguo Zhu ²⁾ , Zhongwei Jiang ³⁾ ¹⁾ Nanjing Normal University, China, ²⁾ University of Technology, Australia, ³⁾ Nanjing University of Aeronautics and Astronautics, China