

# Analytical Optimization for the Reduction of Torque Ripple in Surface Type Brushless AC Motor

Liang Fang, Soon-O Kwon, Sung-Ii Kim, Jung-Pyo Hong  
ECAD Lab., Dept. of Electrical Engineering, Changwon National University  
Sarimdong 9# Changwon, Gyeongnam, 641-773, Korea  
E-mail: [fangliangicw@hotmail.com](mailto:fangliangicw@hotmail.com) Website: <http://ecad.eecu.net>

**Abstract**— An analytical method to optimize the torque performance of permanent magnet brushless AC motors, based on the analytical calculation of the air-gap flux density. From the space harmonic analysis, the flux density, back-EMF and cogging torque can be evaluated with taking account of slots effect in the air-gap magnetic field. To verify proposed analysis method, 2-dimensional finite element analysis is performed and the results are compared. Then, optimization design is performed to reduce of cogging torque through response surface methodology combined with the proposed method.

## I. INTRODUCTION

The air-gap flux density is the determinant factor of the permanent magnet (PM) motor performance, such as back electromotive force (back EMF) and torque characteristics. So, to get the exact flux distribution in the air-gap, an analytical method is used in this paper taking account of the slot-opening effect of a stator.

As be known, cogging torque adds a ripple component to the desired constant output torque from a motor, which can produce vibration and noise that, in turn, reduces the performance of the motor. By using the proved analytical method, we analyzed the cogging torque according to the parameters of the model, such as slot opening width and pole-arc of magnet, which can help us to improve the output torque quality with cogging torque decreased greatly. To verify the analysis results, finite element analysis (FEA) is performed and results are compared.

## II. METHOD OF ANALYSIS

### A. Descriptions of model and assumptions

The analysis model as shown in Fig. 1. is a PM BLAC motor. The specification of the motor is shown in TABLE.1. In order to simplify the field calculation, the following assumptions are made

- (a) the permeability of the iron is infinite;
- (b) the slots are simplified to a rectangular shape;
- (c) the magnetic field distribution is determined from the product of magnetic field produced by magnets and relative permeance;
- (d) magnetic flux in the field across magnets and air-gap in a straight line, wherever a magnet faces a tooth;

### B. Relative permeance

The flux density distribution in the air-gap is non-constant

by taking account of the slots opening effect. The permeance in the slot-opening regions can be expressed as:

$$\lambda = \frac{\mu_0}{g + \frac{h_m}{\mu_r} + \frac{2\pi r_s}{4}} \quad \text{and} \quad \hat{\lambda} = \frac{\lambda}{\left( \frac{\mu_0}{g + h_m/\mu_r} \right)} \quad (1)$$

for  $(k-1)\tau_i - \frac{b_0}{2} \leq x \leq (k-1)\tau_i + \frac{b_0}{2}$

where  $k=1,2,\dots,Q_s$ ,  $(\pi/\tau_p)=p\theta$ ,  $\tau_p$  is the pole pitch;  $\tau_i$  is the slot pitch;  $g$  is the air-gap length;  $h_m$  is magnets height.

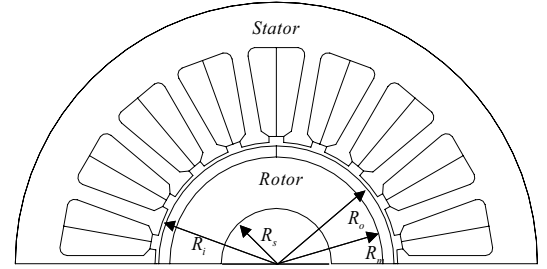


Fig. 1. BLAC Motor Configuration Model

### C. Magnetic field produced by magnets

The magnetic field due to the magnets is obtained from equivalent magnetization and radial component of flux density in air-gap is obtained from Poisson equation[2][3]. The air-gap flux density is given by:

$$B_{magnet} = \sum_{n=1,3,\dots}^{\infty} 4 \frac{M_n}{\mu_r} \cdot \frac{np}{(np)^2 - 1} \left( \frac{R_i}{R_0} \right)^{np-1} \cos np\theta$$

$$\left[ \frac{(np-1)R_0^{2np} + 2R_m^{np+1}R_0^{np-1} - (np+1)R_m^{2np}}{\frac{\mu_r+1}{\mu_r} \{ R_i^{2np} - R_m^{2np} \} - \frac{\mu_r-1}{\mu_r} \{ R_0^{2np} - R_i^{2np} (R_m/R_0)^{2np} \}} \right] \quad (2)$$

where  $n$  is the space harmonic order,  $p$  is the pole pairs,  $\mu_r$  is the recoil permeability of permanent.

The magnetization  $M_n$  is following [2];

$$M = \sum_{n=1,3,\dots}^{\infty} 2M_n \cos np\theta \quad \text{and} \quad M_n = B_r \alpha_p \frac{\sin \frac{n\alpha_p \pi}{2}}{\frac{n\alpha_p \pi}{2}} \quad (3)$$

where  $B_r$  is the remeance, and  $\alpha_p$  is the radio of pole-arc to pole pitch of magnets.

#### D. Calculation of cogging torque

The net lateral force acting on the stator teeth, and hence cogging torque at any rotor position, is calculated by using the non-constant air-gap permeance due to slot-opening effect.

Cogging torque is calculated from:

$$T_c = \sum_{k=0}^{Q_s} l_a \int_0^{b_0/2} \left( \frac{B_{r_{h1}}^2 - B_{r_{h2}}^2}{2\mu_0} \right) r_i dr_s \quad (4)$$

where  $r_i = R_i + r_s$  is the effective radius of cogging torque,  $b_0$  is the slot opening,  $Q_s$  is the number of slots,  $l_a$  is the active length. And the flux density in the slot-opening region is calculated according to assumption (c):  $B_{rb} = B_{magnet} \square \hat{\lambda}$ .

#### E. Comparison

The results get from proposed analytical method are compared with FEA results, as Fig.2 and Fig.3 Show. Comparison of air-gap flux density and cogging torque, the accuracy of the proposed analytical method is well verified. Therefore, we can analysis the characteristics and optimized the performance of the motor using this analytical method.

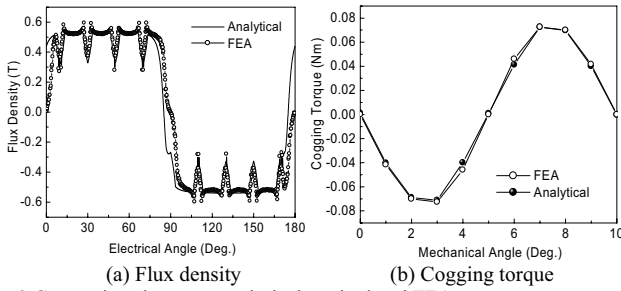


Fig. 2 Comparison between analytical method and FEA

#### III. OPTIMIZATION DESIGN

In this paper, RSM, one of the optimization methods, is used to reduce cogging torque by analytical method, and pole-arc of magnet and slot-opening width are selected as design variables. Table II shows design area considering aspects of cogging torque shown in Fig. 3. Pole-arc of magnet and slot-opening width of an optimal model are 82[°] and 1.156[mm] respectively. At that time, cogging torque of the optimal model is 0.0068[Nm], and it is decreased 95.3[%] than that of the prototype. Fig. 4 describes response surface in design area, and torque performance of the optimal model

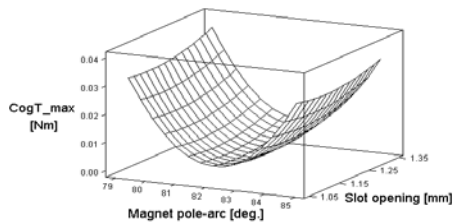


Fig.3 Response surface of optimal model

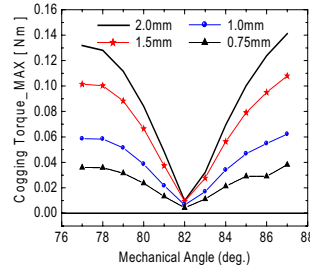


Fig. 4 Cogging torque according to Slot-opening and Pole-arc

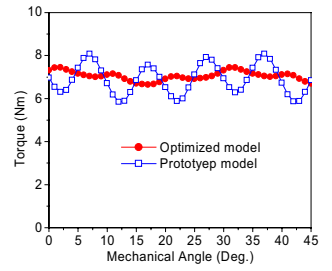


Fig. 5 Output torque of brushless AC motor

and prototype is given in Fig. 5. The torque ripple improved from 32% decreased to 11.3%, for cogging torque greatly reducing by the optimize design.

#### IV. CONCLUSIONS

In this paper, the analytical method considering slot effect of PM motor is used to estimate the flux distribution in the air-gap. The analysis results of air-gap flux density are directly estimated from design parameters. With the flux distribution, back-EMF, and torque characteristics can be calculated and the cogging torque can be greatly improved. The accuracy of space harmonic analysis is verified by good agreements with FEA results. Therefore, the presented analysis method can be useful for initial design stage of PM motor, and optimization design by the method is much more easy than by FEA method.

#### REFERENCES

- [1] T. Li and G. Slemon, "Reduction of cogging torque in permanent magnet motors", *IEEE Trans. Mag.*, vol.24, pp.2901-2903, 1988.
- [2] Z. Q. Zhu, D. Howe, "Analytical prediction of cogging torque in radial field permanent magnet brushless motor" *IEEE Trans. Magn.*, vol 28, pp1371-1374.
- [3] G. H. Kang, J. P. Hong, G. T. Kim, "Analysis of cogging in interior permanent magnet motor by analytical method," *Journal of KIEE.*, 11B, pp 1-8, 2001.
- [4] Z. Q. Zhu, D. Howe, "Effect of slot and pole number on cogging torque in permanent magnet machines," *Proc. of 2<sup>nd</sup> Chinese Int. Conf. On Electrical Machines*, pp.390-394, 1995.

TABLE I  
SPECIFICATION OF ANALYSIS MODEL

ITEMS	SYMBOL	SPECIFICATION(UNITS)
Outer radius of stator		35.5 (mm)
Inner radius of stator	Ri	16.5 (mm)
Outer radius of rotor	Ro	16 (mm)
Inner radius of magnet	Rm	14 (mm)
Radius of shaft	Rs	7.5 (mm)
Length of air-gap	g	0.5 (mm)
Number of slots	Qs	18
Length of stack	la	65 (mm)
Slot opening	bo	2 (mm)

TABLE II  
DESIGN AREA

ITEMS [Unit]	Min.	Max.
Pole-arc of magnet [°]	80	84
Slot opening width [mm]	1.1	1.3