Application of Response Surface Methodology Combined with Experimental Design for Improving Torque Performance of Interior Permanent Magnet Synchronous Motor

Sung-Il Kim[1], Ji-Young Lee[1], Jung-Pyo Hong[1], *Senior Member*, *IEEE*, and Young-Kyoun Kim[2]

[1]Department of Electrical Engineering, Changwon National University

[2]Samsung Electronics Co. LTD.

***Abstract*-- This paper deals with two types of interior permanent magnet synchronous motor (IPMSM), prototype and V-type. One of the both, V-type, is optimized in the rotor shape to obtain better performance than the prototype. Response surface methodology (RSM), as an optimization method, is applied in this paper. In addition, full factorial design (FFD) is used to raise the precision of optimization and reduce the iteration of experiment in the application of RSM. In the end, the usefulness of RSM combined with FFD is verified through the comparison between the fitted response and the results obtained from 2-dimensional finite element analysis.**

I. Introduction

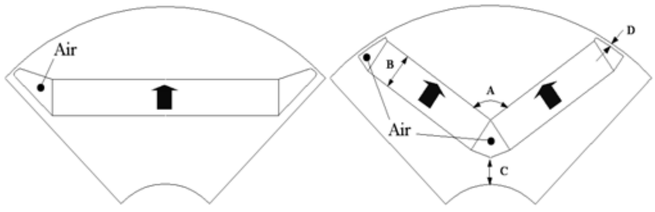
Interior permanent magnet synchronous motor (IPMSM) is one of the most attractive motors applied in compact system. IPMSM has high power density per motor volume because it can utilize both magnet and reluctance torque due to the magnet saliency [1]. However, the torque ripple of IPMSM is relatively large due to generation of the reluctance torque, and it causes noise and vibration. The reluctance torque generally depends on permanent magnet arrangement in the rotor of IPMSM. Therefore, optimization design of the rotor shape is required to improve torque performance of IPMSM [2].

Response surface methodology (RSM) is applied to optimize the rotor shape of IPMSM. It is recently been recognized as an effective optimization approach for design of electrical devices when used in combination of the numerical method for product performance simulation. In RSM, a polynomial model is generally to be constructed to represent the relationship between the performance and the design parameters. Thus, this model can be used to predict the product performance as a function of design variables, and design optimization can be carried out with much ease [3]-[5]. However, the method has a defect varying the precision of optimization according to the size of design region [4]. The defect increases experiment frequency, and brings about a lot of time and cost in the optimization.

As a result, this paper proposes full factorial design (FFD) to solve the problem [6]. That is, responses according to the variation of each design parameter with FFD are examined in the wide design area, and then the narrow design domain to apply RSM is set.

II. Analysis Model and Design Parameters

Fig. 1 shows the configurations of the two analysis models, prototype and V-type. The prototype is used for air-conditioning compressor of an automobile. The V-type is an improved type, applied in this paper, to obtain better the ratio between torque ripple and average operational torque (AOT) than the prototype. Analysis condition of the two models, based on finite element method (FEM), is the same except arrangement and whole volume of permanent magnet (NdFeB) in the rotor.



(a) prototype IPMSM (b) V-type IPMSM

Fig. 1. Rotor configurations of analysis models

The design parameters, from A to D, are described in Fig. 1. (b) : "A", Angle between two segment permanent magnets in the rotor; "B", thickness of permanent magnet; "C", distance from shaft, and "D", link width. Where link width fixes 0.4 [mm] to prevent leakage flux and consider fabrication difficulty.

III. Optimization

A. The Application of FFD for Design Area of RSM

In general, the design area for RSM is set by past experiment data or experimenter's experience. However, if there is none of them, establishment of the domain is very difficult. Furthermore, experimental frequency and the accuracy of optimization are varied according to the dimension of the region. Therefore, 2^3 FFD is used to determine the area in this paper. The advantages of FFD are written as follows [7] :

- All combinations of design parameters are experimented,
- All main and interaction effects of design parameters are evaluated.

Table I shows the array of 2^3 FFD to investigate the ratio between torque ripple and AOT. In Table I, experiment No. 9 is added to estimate the curvature in the middle point of every domain. Main and interaction effects of each parameter are shown in Fig. 2 and Fig. 3 respectively. As an example, main effect of variable A and interaction effect between variable A and B on the ratio are calculated as follows [7] :

- Main effect of A : [sum of all (+1) – sum of all (-1)] / 4
- Interaction effect of AB : [sum of all (+1) – sum of all (-1)] / 4

In conclusion, there is a little both the curvature in the center point and the interaction effects between the parameters. Accordingly, the design domain for RSM is mainly decided by the plots of main effects, and is shown in Table II.

B. The Application of RSM for Optimization

RSM is applied to make an appropriate response model of the ratio in the region given in Table II. A quadratic approximation function of the response model is commonly used to construct the fitted response surface. In general, the response model can be written as follows [8] :

$$\mathrm{Y} = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k} \beta_{ii} x^2_i + \sum_{i \neq j}^{k} \beta_{ij} x_i x_j + \varepsilon \qquad (1)$$

where β is regression coefficients for design variables, ε is a random error treated as statistical error. The observation response vector **Y** at n data point of function Y may be written as matrix notation as

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon} \qquad (2)$$

The least square method, which is to minimize the sum of the squares of the random errors, is used to estimate unknown vector $\boldsymbol{\beta}$. The least square function **L** is

$$\mathbf{L} = \sum_{i=1}^{n} \varepsilon_i^{\,2} = \boldsymbol{\varepsilon}'\boldsymbol{\varepsilon} = (\mathbf{Y} - \boldsymbol{X\beta})'(\mathbf{Y} - \boldsymbol{X\beta}) \qquad (3)$$

Therefore, the estimated vector $\hat{\boldsymbol{\beta}}$ can be written as (4) and the fitted response vector $\hat{\mathbf{Y}}$ is given by (5).

$$\hat{\boldsymbol{\beta}} = (\mathbf{X}'\mathbf{Y})^{-1} \mathbf{X}'\mathbf{Y} \qquad (4)$$

$$\hat{\mathbf{Y}} = \mathbf{X}\hat{\boldsymbol{\beta}} \qquad (5)$$

In this paper, central composite design (CCD) is used to obtain the responses at 15 data point. CCD consists of three portions: a complete 2^k or fractional 2^{k-m} first-order factorial design in which the factor levels are coded into –1 and 1; axial points at a distance α from the center point; one design center point [8]-[9]. The responses as regards the CCD experimental design are obtained by FEM.

It is always necessary to examine the fitted model to ensure that it provides an adequate approximation for the response model. Analysis-of-variance (ANOVA) table shown in Table III is used to confirm precision of the fitted model [9]. In Table III, N is the total number of experiments and k is the number of parameters in the fitted model.

The total variation in a set of data is called the total sum of squares (SST). The quantity SST is computed by summing the squares of deviations of the observed Y_u about their average value $\bar{\mathrm{Y}}$.

$$\mathrm{SST} = \sum_{u=1}^{N} (\mathrm{Y}_u - \bar{\mathrm{Y}})^2 \qquad (6)$$

The SST can be partitioned into two parts, the sum of squares due to regression (or sum of squares explained by the fitted model) and the sum of squares unaccounted for by the fitted. The formula for calculation the sum of squares due to regression (SSR) is

$$\mathrm{SSR} = \sum_{u=1}^{N} (\hat{\mathrm{Y}}_u - \bar{\mathrm{Y}})^2 \qquad (7)$$

The deviation is the difference between the predicted value, $\hat{\mathrm{Y}}_u$, by the fitted model in the uth observation and the overall average of the Y_u.

The sum of squares unaccounted for by the fitted model (SSE) is

$$\mathrm{SSE} = \sum_{u=1}^{N} (\mathrm{Y}_u - \hat{\mathrm{Y}}_u)^2 \qquad (8)$$

The coefficient of determination, R^2, expressed by SST and SSR is

$$R^2 = \frac{\mathrm{SSR}}{\mathrm{SST}} \qquad (9)$$

It is a measure of the proportion of total variation of the values of Y_u about the mean $\bar{\mathrm{Y}}$ explained by the fitted model. A related statistic, called the adjust R^2 statistic is

$$R_A^{\;2} = 1 - \frac{\mathrm{SSE}/(N-k)}{\mathrm{SST}/(N-1)} \qquad (10)$$

It is a measure of the proportion of the estimate of the error variance provided by the residual mean square of the error variance estimate using the total mean square. Accordingly, R^2 and $R_A^{\,2}$ are applied to evaluate accuracy of the fitted model in this paper.

IV. Result and Discussion

The fitted second-order polynomial of V-type IPMSM is expressed as follows :

$$\hat{\mathrm{Y}} = 3152.5 - 41.9\mathrm{A} - 281.2\mathrm{B} - 36.9\mathrm{C} + 0.1\mathrm{A}^2 + 9.5\mathrm{B}^2 + 0.9\mathrm{C}^2 + 1.7\mathrm{AB} + 0.2\mathrm{AC} + 1.8\mathrm{BC} \qquad (11)$$

TABLE I

THE ARRAY OF 2^3 FFD

Experiment No.	A[°]	B[mm]	C[mm]	AB	AC	BC	Ratio[%]
1	100(-1)	3.5(-1)	1.0(-1)	+1	+1	+1	70.0935
2	120(+1)	3.5(-1)	1.0(-1)	-1	-1	+1	32.1670
3	100(-1)	4.5(+1)	1.0(-1)	-1	+1	-1	55.2920
4	120(+1)	4.5(+1)	1.0(-1)	+1	-1	-1	32.3206
5	100(-1)	3.5(-1)	3.5(+1)	+1	-1	-1	72.7231
6	120(+1)	3.5(-1)	3.5(+1)	-1	+1	-1	33.8917
7	100(-1)	4.5(+1)	3.5(+1)	-1	-1	+1	72.3125
8	120(+1)	4.5(+1)	3.5(+1)	+1	+1	+1	27.5542
9	110(0)	4.0(0)	2.25(0)	0	0	0	52.8305

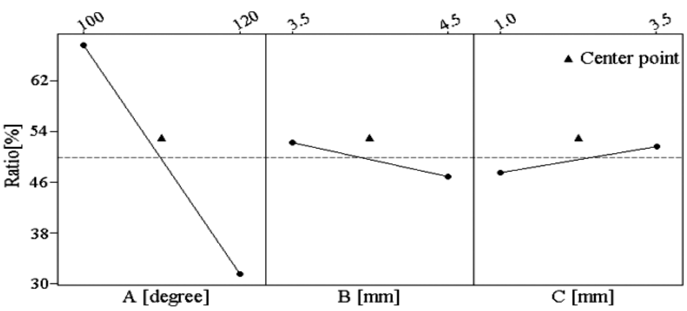


Fig. 2. Main effects of each parameter by 2^3 FFD

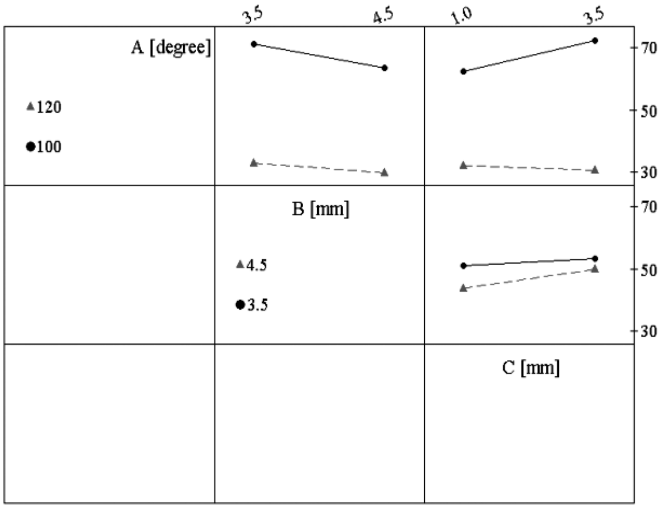


Fig. 3. Interaction effects of each parameter by 2^3 FFD

TABLE II

DESIGN AREA FOR RSM

Design variables	Level of design variables				
	$-\alpha$	-1	0	1	α
A [°]	114.64	116	118	120	121.36
B [mm]	4.23	4.3	4.4	4.5	4.57
C [mm]	0.32	1.0	2.0	3.0	3.68

TABLE III

ANALYSIS OF VARIANCE TABLE

Source of variation	Degree of freedom	Sum of squares (SS)
Regression	k-1	SSR
Residual (error)	N-k	SSE
Total	N-1	SST

Table IV shows the results obtained by FEM and the fitted model in the optimal condition, and the values are almost correspond. Moreover, R^2 and R_A^2 are very high as 0.994 and 0.99 respectively. That is, the fitted model well reflects the responses by FEM in the region given by FFD.

Although the whole volume of permanent magnet is increased 17.5[%], torque ripple is reduced 14.5[%] and AOT is increased 12[%]. The overall ratio between torque ripple and AOT is decreased 8.2[%]. Lastly, RSM combined with FFD finds out the optimal point at a time. Graphic performance comparison between the prototype and the optimized V-type is given in Fig. 4. Response surfaces according to change of the parameters are shown in Fig. 5.

V. CONCLUSION

In order to increase performance of the prototype IPMSM, optimization design by RSM combined with FFD is performed in this paper. The performance of the optimized V-type IPMSM is improved as compared with that of the prototype PMLSM. Therefore, when this proposed approach is applied, it is more efficient to raise the precision of optimization and reduce the iteration of experiment in the optimization design by RSM.

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TABLE IV

THE RESULT COMPARISON IN OPTIMAL POINT

Design variables	Optimal point	Predictive Ratio by RSM	Real Ratio By FEM
A [°]	117.75		
B [mm]	4.23	24.98 [%]	25.23[%]
C [mm]	3.68		

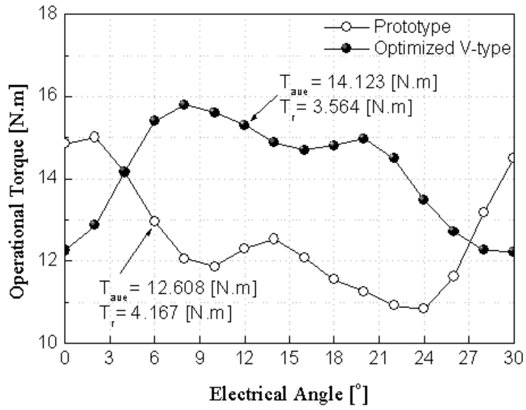
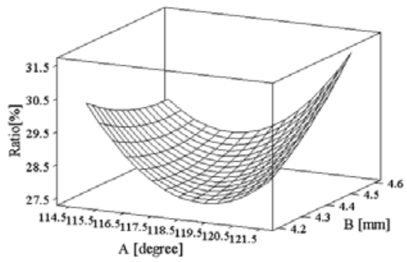
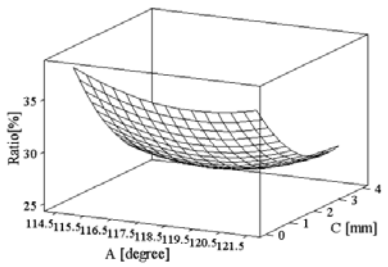


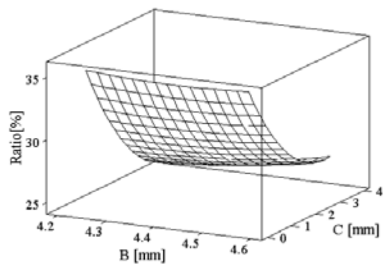
Fig. 4. Performance comparison between prototype and optimized IPMSM



(a) Holding parameter C : 2[mm]



(b) Holding parameter B : 4.4[mm]



(c) Holding parameter A : 118[°]

Fig. 5. Response surfaces of the ratio between torque ripple and AOT

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