

## EQUIVALENT CIRCUIT ANALYSIS OF POLE-CHANGE SINGLE-PHASE INDUCTION MOTOR CONSIDERING HARMONIC COMPONENTS

Hyuk Nam, Seung-Kyu Jung, and Jung-Pyo Hong, Senior Member, *IEEE*

Dept. of Electrical Eng., Changwon Nat'l Univ., Changwon, Gyeongnam, 641-773, Korea  
haeggee@korea.com, namhyuk3@cmpal.com, jphong@sarim.changwon.ac.kr

*Abstract – This paper deals with the characteristic analysis of a pole-change single-phase induction motor using equivalent circuit considering harmonic components, of the magnetic flux density in the air gap. The harmonics can have a significant detrimental effect on the characteristics of the machine such as crawling. Therefore, it is very important to analyze the accurate motor characteristics considering harmonics for many aspects related to the machine design and the performance. In this paper, the magnetic flux density distribution is analyzed by analytical method and Finite Element Method. Discrete Fourier Transform is used to analyze the harmonics and the characteristics are calculated from the equivalent circuit considering the harmonic components. Finally, The characteristic analysis results by the presented method is compared with the experimental results.*

### Introduction

Capacitor-run single-phase induction motors (SPIMs) are widely used in household appliances. The major reason is that the motors are fed directly from the commercial single-phase source without any control devices [1]. A pole-change SPIM in this paper is the capacitor-run SPIM that has two kinds of poles, 4-pole and 2-pole. Therefore, this motor is capable of variable speed operation and can expand the constant torque range using the pole change technique. In addition, it is maintenance-free and cheap in comparison with the motors such as 3-phase inverter motors and brushless DC motors, because it uses a pole change switch to change the speed without inverters or drives. Moreover, when the pole-change SPIM, which can have two outputs and speeds at the same torque using the commercial frequency, is used for the compressor of household appliances, it can improve the system efficiency of the compressor even though the efficiency of the motor itself is a little low. Therefore, the household appliances using the pole change SPIM is expected to be able to have more competitive power in cost and efficiency aspects than that using capacitor-run SPIMs and inverter-type motors.

The pole change SPIM in this paper is composed of a main winding, an auxiliary winding and a compensation winding. The main winding is used at both 4-pole and 2-pole. The auxiliary winding is used only for 4-pole, because it is started at 4-pole. When 4-pole is changed into 2-pole, the main winding is connected with the voltage source but the auxiliary winding is disconnected. At this time, the magnetic flux density distribution by only the main winding can result in severe distortion caused by harmonic components. The existence of harmonics is well known to have a significant detrimental effect on the characteristics of the machine such as crawling [2]. Therefore, to compensate both the magnetic flux density and the torque such as negative torque, the compensation winding is connected in parallel with the main winding at 2-pole operation. However, in spite of the compensation winding, speed-torque curve can be distorted by harmonic components, especially, the third and the fifth harmonics. Therefore, it is very important to calculate the magnetic flux density distribution in the air gap and analyze the characteristics of the pole change SPIM considering the harmonic components for many aspects related to the machine design and the performance.

This paper deals with the characteristic analysis of the pole-change SPIM using equivalent circuit considering harmonic components of the magnetic flux density in the air gap. The magnetic flux density distribution is analyzed by analytical method and Finite Element Method (FEM). Discrete Fourier Transformation (DFT) is used to analyze the harmonics in the magnetic flux density distribution and the harmonics are applied to calculate the characteristics using the equivalent circuit. Finally, the characteristic analysis results by the presented method are compared with the experimental results.

### Pole-Change Technique

Fig. 1 shows the winding patterns at 4-pole and 2-pole operations. When pole is changed from 4-pole to 2-pole, pole  $N'$  and pole  $S'$  of the main winding in Fig. 1(a) are changed into pole  $S''$  and pole  $N''$  in Fig 1(b), respectively. The auxiliary winding is disconnected and the compensation winding is connected in parallel with the main winding instead of the auxiliary winding as shown in Fig. 1(c).

Fig. 2 shows the magnetic flux density distribution by FEM, DFT of the magnetic flux density distribution and the speed-torque curves, which are obtained from the experimental results according to the existence of the compensation winding. 90 deg. of rotor position in Fig. 2(a) corresponds to the pole center at 2-pole.

In Fig. 2(a) and (b), the unbalanced 6-pole of N-S-N-S-N-S occurs due to the harmonic components such as the third and the fifth order at 2-pole operation with only the main winding. As the result, the third harmonic component synchronizes the speed near 1,200 rpm, and the speed-torque curve is distorted and generates the negative torque, which is larger than the positive torque as shown in Fig. 2(c). Thus, the compensation winding is wound to compensate both the magnetic flux density distribution and torques such as the negative torque and the maximum torque at 2-pole.

As shown in Fig. 2, the magnetic flux density distribution of 6-pole is changed into that of 2-pole by the compensation winding, and the harmonic components as well as the negative torque are reduced. However, in spite of the compensation winding, the distortion of the speed-torque curve can be still produced. Therefore, it is very important to calculate the magnetic flux density distribution in the air gap and analyze the characteristics of the pole change SPIM considering the harmonic components.

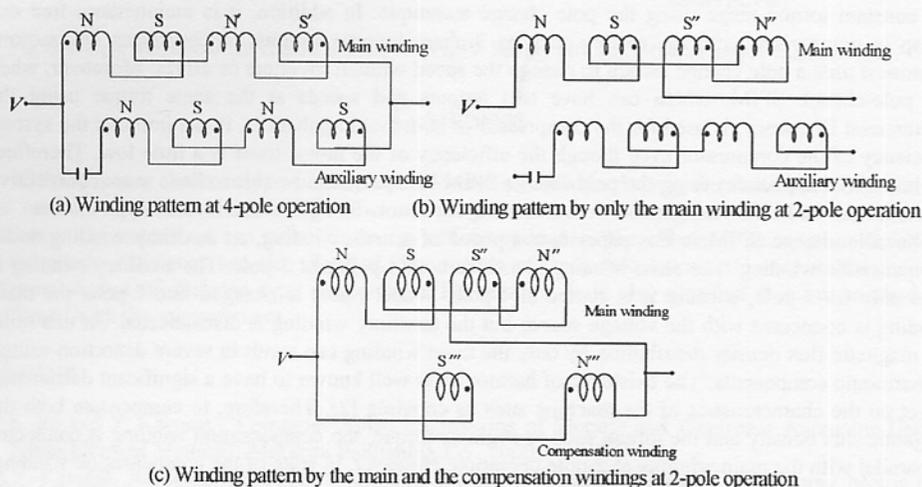
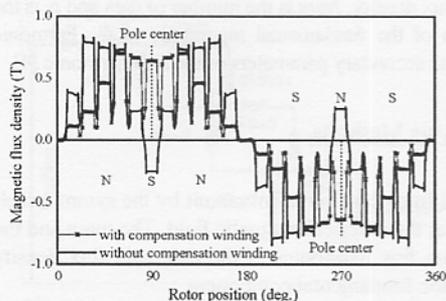
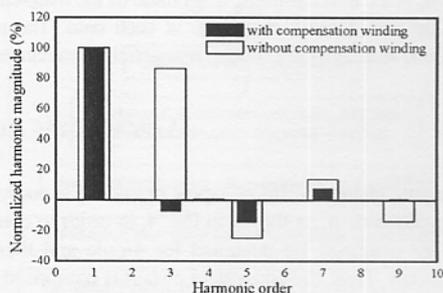


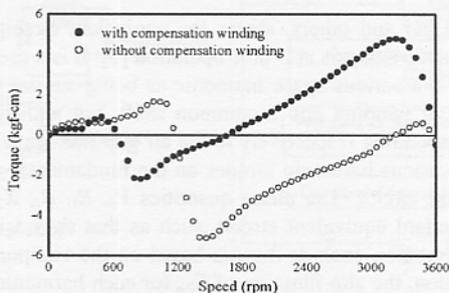
Fig. 1 Winding pattern according to the pole number



(a) Magnetic flux density distributions



(b) DFT of the magnetic flux density distribution



(c) Experimental results of the speed-torque curves

Fig. 2 Characteristics according to the compensation winding at 2-pole operation

## Analysis Methods

### Magnetic Flux Density in the Air Gap and DFT

The magnetic flux density distribution by analytical method is calculated as follows. To avoid unnecessary mathematical complexity, certain simplifying assumption is made. The main simplifying condition is the assumption of the infinite relative permeability of iron ( $\mu_r = \infty$ ). It is assumed that the motor consists of two smooth coaxial cylinders made of a magnetic material and the cylinders are separated by the air gap in Fig. 3.

The magnetic flux density produced by an arbitrary system of conductors in the air gap is obtained by the superposition of the field densities of the individual conductors. The magnetic flux density  $B(\alpha)$  of the individual turns at any point  $P$  having the coordinate  $\alpha$  is obtained by using equation (1) [3].

$$B(\alpha) = \frac{\mu_0 C_\theta}{\pi \delta} \sum_{n=1}^{\infty} \frac{1}{n} \sin n(\alpha - \theta) \quad (1)$$

where  $n$  is the harmonic order,  $C_\theta$  is the number of conductors that are placed at the position  $\theta$  and  $\delta$  is the magnetic air gap.

DFT for the harmonic analysis of the magnetic flux density distribution can be expressed as equation (2).

$$a_n = \frac{2}{Num} \sum_{i=0}^{Num} b_i \sin \left( \frac{2\pi ni}{Num} \right) \quad (2)$$

where  $a_n$  is the  $n$ -th harmonic magnitude of the magnetic flux density,  $Num$  is the number of data and  $b_i$  is the magnetic flux density magnitude at each data. The ratio of the fundamental magnitude to the harmonic magnitude  $a_n/a_1$  is used to obtain magnetizing reactances and secondary parameters of the  $n$ -th harmonic [4].

### Characteristic Analysis Methods

The pole change SPIM at 4-pole operation is analyzed from the equivalent circuit by the symmetrical-coordinate system, as shown in Fig. 4, in order to consider the elliptical magnetic field. The main and the auxiliary windings are designed for 4-pole and there are few harmonics in the magnetic flux density distribution. Thus, the equivalent circuit is composed of the fundamental component.

On the other hand, the harmonics should be considered at 2-pole, because the 2-pole characteristics affected by the harmonic.

Fig. 5, as suggested by Alger and others, shows the equivalent circuit considering the harmonic components for the characteristic analysis at 2-pole operation [5]. It is a useful concept to visualize the electromagnetic behavior of the various space harmonic as being similar to the behavior of separate motors, with a common stator winding and a common shaft, but with magnetizing reactances and secondary impedances corresponding respectively to the air gap flux wave of each specific harmonic. Therefore, the effect of the various harmonic torques on the fundamental speed-torque curves can be evaluated from the equivalent circuit. The phase quantities  $V_1$ ,  $R_1$ ,  $R_2$ ,  $X_2$  and  $X_M$  of the circuit are identical to those of the standard equivalent circuit, such as that shown in Fig. 5. The magnetizing reactance of each of the harmonics, such as  $X_{M,5}$ , is based on the component of air gap flux of that particular harmonic. In addition, the slip function of  $R_{2,n}$  for each harmonic is set up for the rotor slip for that particular harmonic and is dependent on the order of the harmonic and on whether the harmonic field is positive (or forward) rotating or negative (or backward) rotating.

Thus, the  $n$ -th slip function can be expressed as equation (3) if it is a forward rotating field and it can be expressed as equation (4) if it is a backward rotating field [5].

$$s_{pn} = 1 - n(1 - s) \quad (3)$$

$$s_{nn} = 1 + n(1 - s) \quad (4)$$

where  $n$  is the order of the harmonic,  $s$  is the slip of the fundamental component. Considering the fundamental component as a harmonic of the first order, the harmonic order of the pole change SPIM consists of odd terms only.

Fig. 6 shows the flow chart for the characteristic analysis at 2-pole operation. The phases of the main and the compensation windings are same. Therefore, the mutual effect on the two windings should be considered. Moreover, the characteristics of each winding are calculated by using the equivalent circuit of Fig. 6, respectively. The total torque can be obtained from superposition of the torques by the main and the compensation windings according to slip.

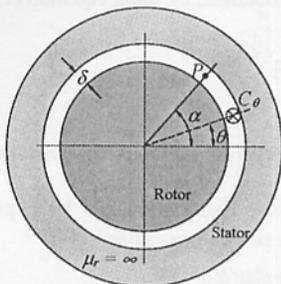


Fig. 3 Illustration relating to the calculation of the field of conductors

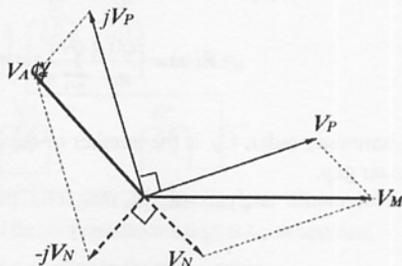


Fig. 4 Symmetrical-coordinate system

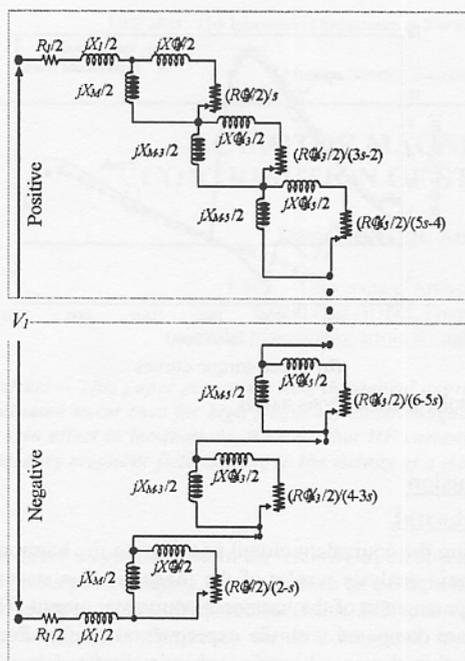


Fig. 5 Equivalent circuit considering the harmonics

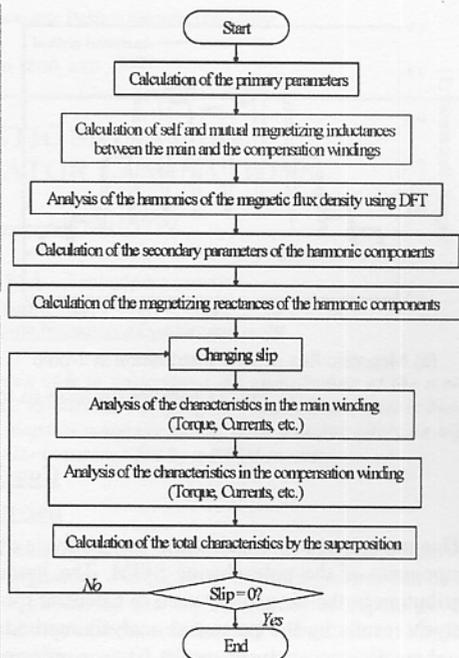


Fig. 6 Flow chart for the characteristic analysis

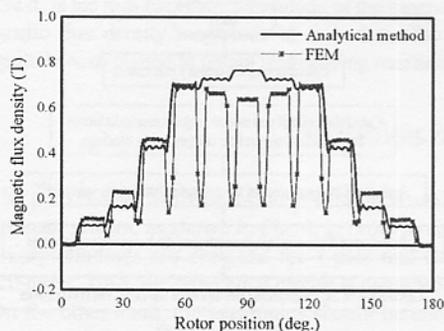
## Analysis and Experimental Results

To verify the presented analysis method in this paper, the characteristics are analyzed using the analysis model of which the specifications is input voltage of 220 V, frequency of 60 Hz and rated torque of 4.4 kgf-cm. While output power and synchronous speed are 160 W and 3,600 rpm at 2-pole operation, those of 4-pole operation are 80 W and 1,800 rpm, respectively.

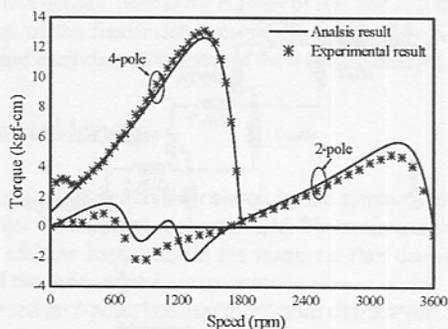
Fig. 7(a) shows the magnetic flux density distribution in the air gap using the analytical method in comparison with the FEM. The effect on the saturation can be considered by the FEM, but not by the analytical method. Therefore, the difference between the harmonic analysis results by two methods occurs as shown in Table. 1. As the results, the speed-torque curve of the analysis model by the experimental result is affected by the fifth order harmonic component, while that of the analysis model by the simulation result is more affected by the third order harmonic component as shown in Fig 7(b). Even though the speed-torque curve of the analysis result differs from that of the experimental result, the analysis results by the presented method show the effect of the harmonics and the important torques such as the pole change torque and the maximum torque at 2-pole can be estimated. Thus, this presented method is used for the characteristic analysis.

Table. 1 Harmonic analysis results

Harmonic order	Analytical method (%)	FEM (%)
1	100.0	100.0
3	-17.6	-7.5
5	-7.3	-14.5
7	4.0	7.5



(a) Magnetic flux density distribution at 2-pole



(b) Speed-torque curves

Fig. 7 Characteristic analysis results of the analysis model

## Conclusion

This paper presents the characteristic analysis using the equivalent circuit considering the harmonic components of the pole change SPIM. The harmonic analysis results of the magnetic flux density distribution in the air gap are used to calculate the parameters of the harmonic equivalent circuit. The analysis results by the presented analysis method are compared with the experimental results. From the characteristic analysis results, it is confirmed that the speed-torque curve is distorted by the harmonic components, especially, the third and the fifth harmonics, contained in the magnetic flux density distribution. Therefore, there is need to design the windings in order to reduce the harmonics and obtain torques such as pole change torque and breakdown torque at 2-pole operation.

## References

- [1] H. Nam, K. H. Ha, J. J. Lee, J. P. Hong, and G. H. Kang, "A study on iron loss analysis method considering the harmonics of the flux density waveform using iron loss curves tested on Epstein samples," *IEEE Trans. Magn.*, Vol. 39, No. 3, pp. 1472-1475, May 2003.
- [2] H.A. Toliyat, and N. Sargolzaei, "A Comprehensive Method for Transient Modeling of Single Phase Induction Motors Including the Space Harmonics," *Journal of Electric Machines and Power Systems*, Vol. 26, No. 3, pp. 221-234, 1998.
- [3] B. Heller and V. Hamata, *Harmonic Field Effects in Induction Machines*, Academia, Publishing House of the Czechoslovak Academy of Sciences, 1977.
- [4] C. G. Veinott, *Theory and Design of Small Induction Motors*, McGRAW HILL, 1959.
- [5] P. L. Cochran, *Polyphase Induction Motors, Analysis, Design, and Application*, Marcel Dekker, Inc., 1989.



XI INTERNATIONAL SYMPOSIUM  
ON **ELECTROMAGNETIC FIELDS**  
IN ELECTRICAL ENGINEERING  
**ISEF 2003**



SEPTEMBER 18-20, 2003

MARIBOR, SLOVENIA

**The Symposium is organised by:**

- Research group Applied Electromagnetics, Faculty of Electrical Engineering and Computer Science, Maribor, Slovenia
- University of Maribor, Slovenia
- Institute of Mechatronics and Information Systems, Technical University of Lodz, Poland
- Department of Fundamental Research, Electrotechnical Institute, Warsaw, Poland

VOLUME 1

5.	<b>Simulation of 3D End Effects in a Superconductive Magnet with a Ferromagnetic Core</b> H. De Gersem, T. Weiland.....	347
6.	<b>Modeling of Ferromagnetic Using Classical and Domain Models</b> Z. Gmyrek, G. Zwoliński.....	353
7.	<b>Modeling of Flux Concentrator's Operation in the Magnetic Forming System</b> R. Goleman, K. Nalewaj.....	359
8.	<b>FDTD with Magnetic Nonlinearity</b> A. Iványi, P. Kis.....	363
✓9.	<b>Development of AC Reactors by Using Metal Powder Materials</b> G.H. Lee, Y.K. Kim, J.Y. Lee, J.P. Hong, K.H. Ha, K.M. Yoon.....	369
10.	<b>Soft Magnetic Composite in Design of Motor With Oscillatory Starting</b> D. Miljavec, B. Šuštaršič, Ž. Turk, K. Lenasi.....	375
11.	<b>Permanent Magnets Dealt with Boundary-Integral Approach</b> K. Pawluk.....	381
12.	<b>Scaling of Magnetization Characteristics in Pspice Program</b> Z. Włodarski, J. Włodarska, A. Brykalski.....	387
13.	<b>Designed Giant Magnetoresistances of Ni-Mn and Fe-Si Alloy System in High Fields up to 30T</b> K. Yamada, Z. Honda, K. Miyake, T. Okazaki, Y. Sakisaka, Q. Xie.....	393

#### ORAL SESSION:

*ELECTROMAGNETIC ENGINEERING 3 (EE-3, Computational Techniques) [11.15-13.15]*

*Chairman: Prof. T. Tsiboukis*

1.	<b>Analytical Computation of the Electromagnetic Torque in Doubly Slotted Synchronous Machines with Distributed Windings</b> A. Di Gerlando, G.M. Foglia, R. Perini.....	399
2.	<b>Analytical Evaluation of the Electromotive Forces in Doubly Slotted Synchronous Machines with Distributed Windings</b> A. Di Gerlando, G.M. Foglia, R. Perini.....	405
3.	<b>A Study on Improvement in Energy Efficiency of Skeleton Type Single Phase PM Motor Using Finite Element Method</b> T. Masuda, Y. Takashima, M. Murase, Y. Kawase, T. Yamaguchi, T. Okouchi.....	411
✓4.	<b>Equivalent Circuit Analysis of Pole-Change Single-Phase Induction Motor Considering Harmonic Components</b> H. Nam, S.K. Jung, J.P. Hong.....	415
5.	<b>AC Motor Magnetic Signature: Contribution of Stator Laminations</b> D. Roger, A. Henneton.....	421