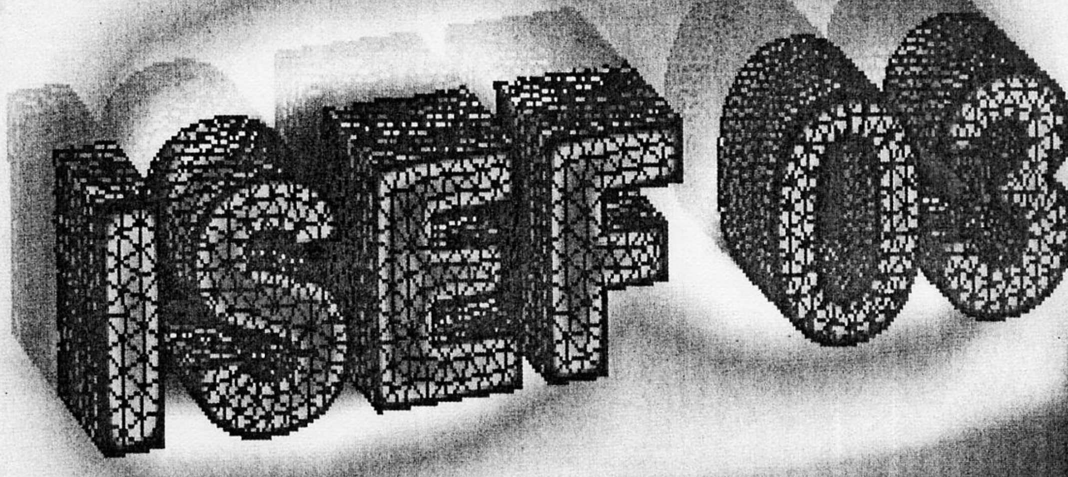
# XI INTERNATIONAL SYMPOSIUM ON **ELECTROMAGNETIC FIELDS** IN ELECTRICAL ENGINEERING

## ISEF 2003




SEPTEMBER 18-20, 2003

MARIBOR, SLOVENIA

**The Symposium is organised by:**

- Research group Applied Electromagnetics, Faculty of Electrical Engineering and Computer Science, Maribor, Slovenia
- University of Maribor, Slovenia
- Institute of Mechatronics and Information Systems, Technical University of Lodz, Poland
- Department of Fondamental Research, Electrotechnical Institute, Warsaw, Poland

VOLUME 2

# OPTIMIZATION OF BARRIER TYPE SRMs with RESPONSE SURFACE METHODOLOGY COMBINED WITH MOVING LEAST SQUARE METHOD

[1]Young-Kyoun Kim, [1]Ji-young Lee, [1]Jung-Pyo Hong, Senior Member, *IEEE*, and [2]Jin Hur

[1]Dept. of Electrical Eng., Changwon Nat'l Univ., Changwon, Kyungnam, 641-773, Korea
[2]Precision Machinery Research Center of Korea Electronic Technology Institute, Korea
( ensigma@hitel.net, jyecad@korea.com, jphong@sarim.changwon.ac.kr, jinhur@keti.re.kr )

Abstract – *While Switched Reluctance Motors have good performances, such as high torque/volume ratio, high speed, and high reliability, the motor has serious disadvantage on large torque ripple. In this paper, therefore, the Switched Reluctande Motor having barriers in the rotor is proposed to improve the torque performance of the motor. In order to reduce the torque ripple, the shape optimization of the barrier is accomplished by a combination technique employing Response Surface Methodology, Moving Least Square Method, and Genetic Algorithm.*

## Introduction

The applications of switched reluctance motors are recently received much attention in various industrial fields due to its superior advantages, such as high speed, efficient variable speed, and high reliability [1-2]. However, the SRM has severe nonlinear characteristics and suffers from high torque ripple and the torque/volume ratio dependents on the designed shape of salient poles, which lead to limit their applications in industrial fields. Therefore, in order to improve the torque performance, this paper introduces a SRM having barriers [3] in the rotor and describes the optimum design of the barrier shape.

A computational optimization can be complicated and time-consuming because the numerous design variables are used and have usually a lot of interactions each other. Therefore, in order to introduce an effective computational approach, the Response Surface Methodology (RSM) is employed [4-6]. The RSM is used to build an analytical expression, which is utilized as an objective function or constraint in the optimization process. Moreover, to enhance the accuracy of the analytical model, the regression coefficients in the RSM are estimated by using the Moving Least Square (MLS) method [7]. The RSM combined with the MLS method is able to display an overall perspective of the torque performance according to the behavior of design variables, which are the components of the barrier shape. Although, several different search methods can be available for the optimization problem, Genetic Algorithms [8] have been selected to use as the search method because the genetic algorithms are able to find the global optimum. And the electromagnet field within the motor is computed by using the time-stepping procedure, which is coupled with 2D-FEM and the voltage equation together.

## Concept of the Statistical Fitting Method

### Response Surface Methodology

The RSM is very powerful method, which seeks the relationship between input variable and output variable through statistical fitting method, to make simple model for a complex problem. A polynomial approximation model is commonly used for a second-order response and can be written as following [4-6].

$$U = \beta_0 + \sum_{j=1}^{k} \beta_j x_j + \sum_{j=1}^{k} \beta_{jj} x_j^2 + \sum_{i \neq j}^{k} \beta_{ij} x_i x_j + \varepsilon \tag{1}$$

where $\beta$ is regression coefficients and $\varepsilon$ denotes the random error. The least squares method is used to estimate unknown coefficients. The least squares method, which is to minimize the sum of the squares of the random errors, is used to estimate unknown vector $\boldsymbol{\beta}$. The least squares function can be expressed as follows:

$$L = \sum_{i=1}^{n} e_i^2 = e'e = (U - X\beta)'(U - X\beta) \tag{2}$$

The estimated vector $\hat{\beta}$ of the unknown vector $\beta$ must satisfy as (3).

$$\left.\frac{\partial L}{\partial \beta}\right|_{\beta} = -2X'U + 2X'X\beta = 0 \tag{3}$$

Therefore, the estimated vector $\hat{\beta}$ can be written as (4) and the fitted response vector $\hat{\mathbf{U}}$ is given by (5). Matrix notations of the fitted coefficients and the fitted response model should be

$$\hat{\beta} = (X'X)^{-1}X'U \tag{4}$$

$$\hat{U} = X\hat{\beta} \tag{5}$$

## Moving Least Square Method

In the traditional regression, coefficients of the functional approximation are estimated by the Least Square (LS) method. The LS method has a weak point increasing approximation errors in some cases because of minimizing the sum of the squares of the errors through whole sample points. To overcome its disadvantage, the MLS method is proposed to fit relationship between inputs and outputs. Unknown coefficients of the functional response modeling are estimated by the MLS method. The main idea of the MLS method is that a whole approximation $U^h(x)$ of a sampling space can be accomplished by going through a moving process.

In domain W, an unknown relationship between inputs and outputs can be expressed as follows:

$$U(x) = U^h(x) + \varepsilon \tag{6}$$

where $\varepsilon$ denotes the random error mean zero and variance $\sigma^2$, $U^h(x)$ is an approximation function in the unknown relationship, and $x$ is an design variable.

$$U^h(x) = \sum_{i=1}^{m} P_i(x)\, a_i(x) = \mathbf{P}(x)^T\, \mathbf{a}(x) \tag{7}$$

where $m$ is the number of the polynomial basis, $P_i(x)$ is a complete polynomial basis, and $a_i(x)$ is unknown coefficients.

The coefficients can be estimated by minimizing a weighted least square function, which is shown as follows:

$$J(d) = \sum_{I}^{N} W_I(d)\, \varepsilon_I^2 = \sum_{I}^{N} W_I(d) \left(U(x_I) - U^h(x_I)\right)^2 \tag{8}$$

where $W_I(d)$ is a weight function depend on the distance ($d$) between approximation point and point I in a influence domain and $N$ is the number of points in that.

In this work, the quartic spline function is used as the weight function [7].

$$W(d) = \begin{cases} 1-6\left(\frac{d}{r}\right)^2+8\left(\frac{d}{r}\right)^3-3\left(\frac{d}{r}\right)^4 & \text{for } \frac{d}{r} \le 1 \\ 0 & \text{for } \frac{d}{r} > 1 \end{cases} \tag{9}$$

where $r = \alpha\, d_I$ is the radius of influence domain and a is the scaling factor.
Substituting (7) and (9) into (8), the matrix notation of the unknown coefficients can be rewritten as follows:

$$\mathbf{a}(x) = \mathbf{A}^{-1}\mathbf{B}\,\mathbf{U} \tag{10}$$

$$\mathbf{A} = \mathbf{P}^T\,\mathbf{W}\,\mathbf{P}\,,\ \mathbf{B} = \mathbf{P}^T\,\mathbf{W} \tag{11}$$

where **P** and **U** are composed with a set of sample points regarding design variables and outputs, respectively.

## Application to the Optimization Design in Barrier Type SRM

### Barrier Type SRM and Field computation Method

The proposed barrier type 8/6 SRM is shown in Fig. 1 and the barrier is employed to improve the torque characteristic. The voltage equation of the motor is written as follows:

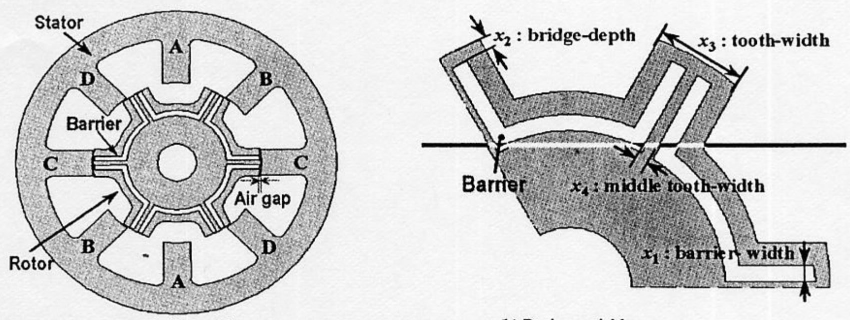
$$V_a = R_a i_a + L_a \frac{d i_a}{d t} + E_a \tag{12}$$

where $E_a$ denotes the motor back emf, $R_a$ is the resistance of windings, and $L_a$ is inductance of that, per a phase. The governing equation of the electromagnetic filed in the motor can be described by magnetic vector potential $\vec{A}$.

$$\nabla \times [\nu(\nabla \times \vec{A})] = \vec{J} \tag{13}$$

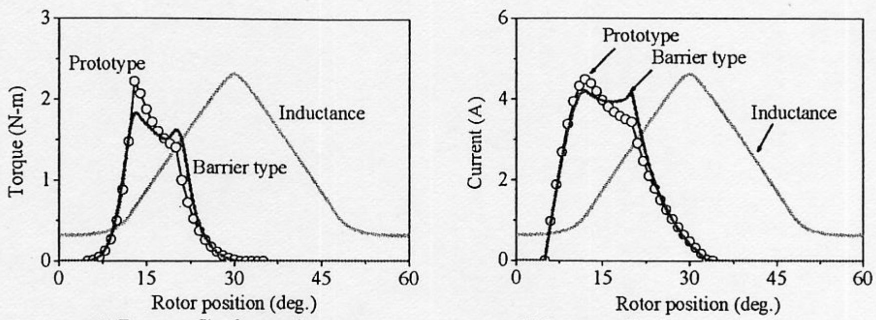
where n is the magnetic reluctivity and $\vec{J}$ is the *z*-component of the density of the current.

To consider the nonlinear nature of inductance as a function of exciting current and rotor angular displacement, it is necessary to solve the differential circuit equations for the appropriate switched conditions and transient performance. Therefore, system matrix is composed of coupling the voltage equation (1) with the governing equation (2), and solved by the time-stepping procedure. The reluctance torque is calculated by using the Maxwell stress tensor.




(a) Analysis model. (b) Design variables.

Fig. 1. Analysis model and design variables.




(a) Torque profile of one stroke. (b) Current profile of one stroke.

Fig. 2. Analysis results of the barrier type SRM.

When an input voltage is 150 V and a speed is 1800 rpm, Fig. 2 shows the comparison of analysis results of the prototype SRM and the barrier type SRM. The analysis results are computed by time-stepping procedure, and as will be seen, the reduction of the torque ripple is achieved in the barrier type SRM.

## Objective function and Procedure of Optimization

While several different optimization methods can be applied at the motor design, the optimization technique based on the genetic algorithms has been used in this paper. The genetic algorithms research in the design space of the motor by means of a process of reproduction, crossover and mutation [8], which is able to avoid the local minimum as contrasted with gradient based method.

The schematized procedure of the optimization is shown in Fig. 3. In order to apply the shape optimization of the barrier, an objective function is defined as the torque ripple of the barrier type SRM and the constraint condition is defined to satisfy the running torque of the prototype SRM as follows:

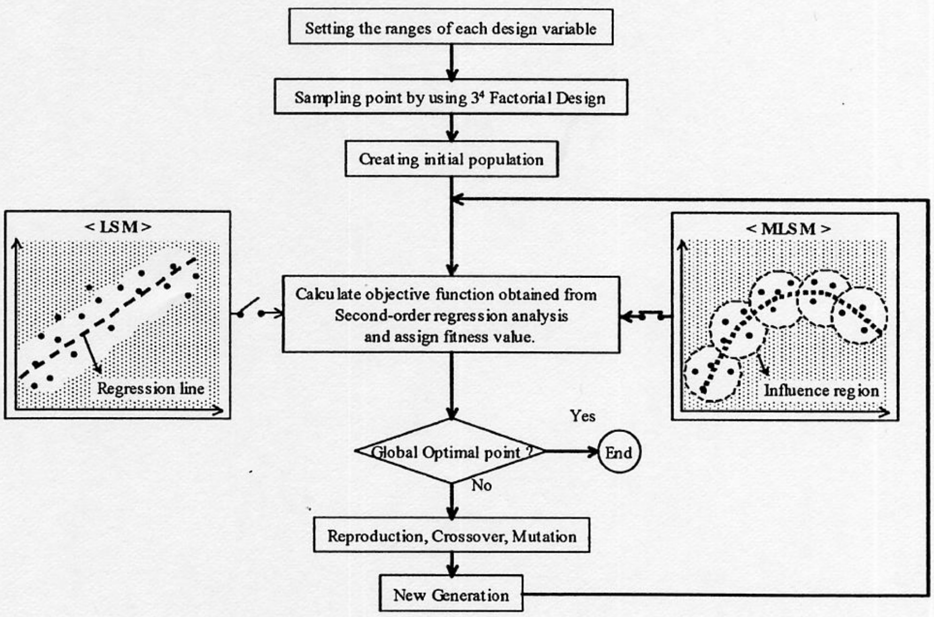



Fig. 3. Description of Optimization Based on RSM associated with Genetic Algorithms.

Minimize:

$$f(x) = U^h(x)_{Torque\ ripple}\ \text{(N - m)} \tag{14}$$

Subject to:

$$g(x) = U^h(x)_{Torque}\quad \text{(N - m)} \geq 1.37 \tag{15}$$

Design space:

$$0.6 \leq x_1 \leq 2.2,\ 0.3 \leq x_2 \leq 1.1,\ 18 \leq x_3 \leq 20,\ 1.2 \leq x_4 \leq 2.4 \tag{16}$$

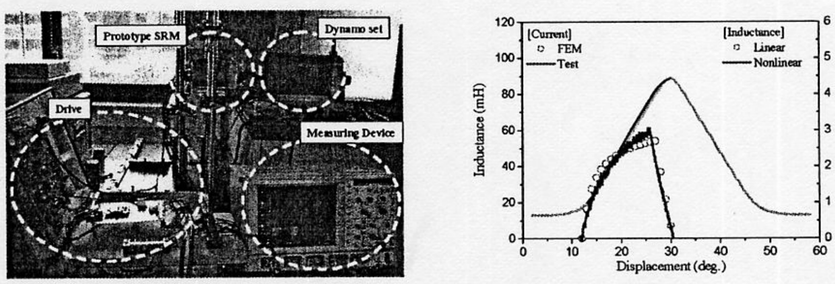
In this paper, the functional expressions of the objective function and constraint are obtained by the sampling point based RSM combined with MLS and the full factorial design is used for building the second-order fitted model. This design of experiments involving four factors is required to conduct 81 experiments. And then the time-stepping procedure, which is coupled with 2D-FEM and the voltage equation together, is used for acquiring the samples.

## Results of Numerical Optimizaton and Discussion

Fig. 4(a) shows the overall experimental test-bed, built at Changwon University in Korea and it was tested under the conditions for experiment with 80V input voltage and 800 rpm speed. As shown in Fig. 4(b), it is noted that the simulation results has a good agreement with the experimental results.

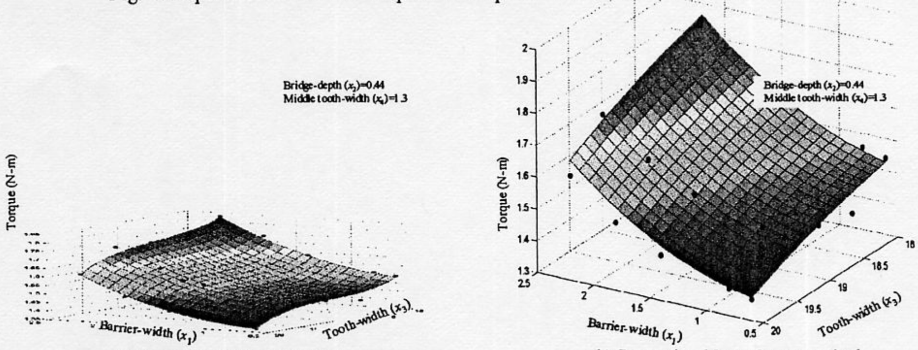
Figs. 5 and 6 show the perspective response surface of the torque characteristics that have been fitted with 81 samples where solid black dots represent the simulation results of 2D-FEM. From these results, it can be confirmed that the MLSM gives a response surface, which is much closer to the results of 2D-FEM.

In this paper, The RSM combined with the genetic algorithms is applied to the optimum design of the barrier shape in the proposed barrier type 8/6 SRM. The design parameters of the proposed motor and torque performances are shown in Table 1. The torque ripple of the proposed motor is less than that of the prototype SRM and its running torque becomes greater than that of the prototype SRM.



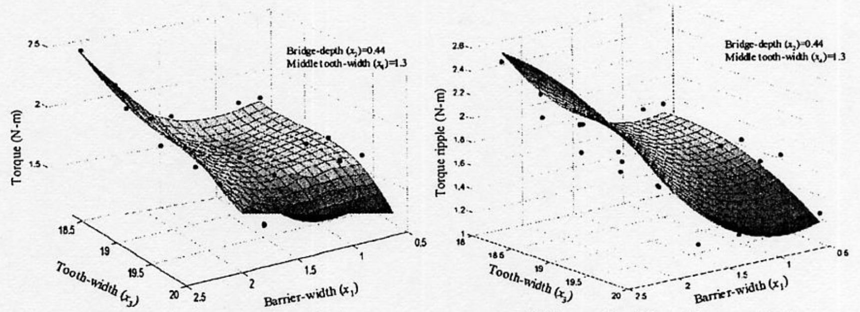

(a) Experimental set. (b) Comparison of phase current.

Fig. 4. Experimental set and comparision of phase current about prototype SRM.




(a) Moving Least Square method. (b) Conventional Least Square method.

Fig. 5. Response surface of torque with both fixed $x_2$ and $x_4$.




(a) Moving Least Square method. (b) Conventional Least Square method.

Fig. 6. Response surface of torque ripple with both fixed $x_2$ and $x_4$.

Table 1. Analysis model and design variables.

| Item | Prototype SRM | Barrier type SRM |
|---|---|---|
| Torque | 1.37 (N-m) | 1.4 (N-m) |
| Torque ripple | 1.65 (N-m) | 1.27 (N-m) |

**Optimized shape of barrier in the proposed motor**

| Barrier-width ($x_1$) | Bridge-depth ($x_2$) | Tooth-width ($x_3$) | Middle tooth-width ($x_4$) |
|---|---|---|---|
| 1.0 (*mm*) | 0.44 (*mm*) | 20 (deg.) | 1.3 (mm) |

## Conclusion

In this paper, the Barrier Type SRM is proposed in order to improve the torque performances. Also, taking advantages of barrier shape, the optimization technique based on the statistical ffiting method has been developed. Moreover, Moving Least Square method is introduced to enhance the accuracy of the predictive performance according to varieties of design parameters. The comparison with the prototype SRM has been performed and the result of the comparison indicates that the torque performance of the Barrier type SRM is superior to that of the prototype SRM.

REFERENCES

[1] M. stiebler and Ke Liu, "An Analytical Model of Switched Reluctance Machines," *IEEE Trans. Energy Conversion, Magn.*, vol. 14, No. 4, pp. 1100-1107, December 2000.

[2] Yasuharu Ohdachi, Yoshihiro Kawase, Yutaka Miura, and Yoji Hayashi, "Optimum Design of Switched Reluctance Motors using Dynamic Finite Element Analysis," *IEEE Trans. On Magnetics*, Vol. 33, No. 2, pp. 2033-2036, March 1997.

[3] Hiroyuki Kiriyama, shinichiro Kawano, Yukio Honda, Toshiro Higaki, Shigeo Morimoto, and Yoji Takeda, "High Performance Synchronous Reluctance Motor with Multi-Barrier for the Appliance Industry," *IEEE Industry Application Conference*, pp. 111-117, 1998.

[4] R. Rong, D. A. Lowther, Z. Malik, H. Su, J. Nelder, and R. Spence, "Applying Response Surface Methodology in the Design and Optimization of Electromagnetic Devices," *IEEE Trans. On Magnetics*, Vol. 33, No. 2, pp. 1916-1919, MARCH 1997.

[5] F. Gillon and P.Brochet, "Screening and Response Surface Method Applied to the numerical optimization of electromagnetic devices," *IEEE Trans. On Magnetics*, Vol. 36, No. 4, pp. 1163-1167, JULY 2000.

[6] R. H. Myers and D. C. Montgomery, *Response Surface Methodology*, John wiley & Sons, INC., 1995.

[7] Vlatko Čingoski, N. Miyamoto, and H. Yamashita, "Element-Free Galerkin Method for Electromagnetic Field Computations," *IEEE Trans. On Magnetics*, Vol. 34, No. 5, pp. 3236-3239, 1998.

[8] N. Bianchi and S. Bolognani, "BRUSHLESS DC MOTOR DESIGN: AN OPTIMISATION PROCEDURE BASED ON GENETIC ALGORITHMS," *EMD97 Conference Publication*, No. 444, pp. 16-20, September 1997.