Torque Characteristics Analysis Considering the Tolerance of Electric Machine by Stochastic Response Surface Method

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Abstract—**Manufacturing tolerances as well as measuring errors have a great influence on products designed by optimization technique etc. to improve its characteristics and reduce the cost. Therefore, tolerance analysis technique is required to find the tolerance band of design variables and estimate the characteristic distribution of the products. In this paper, we introduce Stochastic Response Methodology (SRSM), which treated design variables as random ones. Also, we present a way of the tolerance analysis from electric viewpoint. A BLDC motor is analyzed to verify the application of this method. Finally, the validity of this method is confirmed by obtaining statistically significant results for torque characteristics of the motor.**

Keywords—optimization; stocastic response surface method; tolerance analysis; BLDC motor

I. INTRODUCTION

Facing the rising cost of electric energy, the motor users and manufactures have begun to pay attention to highly efficient motor in an attempt to reduce their costs. These requirements are mainly achieved by optimizing motor designs. The optimization of the electric machine is believed to be the most economical approach to improve the motor efficiency and performance. Although the electric motor is designed by using optimization algorithm, its performance cannot be satisfied as the desired one in certain cases. This occurs because the approaches of optimization neglect the effects on variation of design variables such as manufacturing tolerances.

The design of the electric machine needs allowance for dimensional tolerances due to limitations on the manufacturing and measuring precision on every part: for example, tolerances occur on stator and rotor punching, frame dimensions, bearing clearances, magnetic and electric material properties, and so on [1]. In general, the larger tolerance in manufacturing processes lead to have an influence on the lower cost of manufacturing machines. These dimensional tolerances, however, can effect on the electric machine performance, such as operating efficiency, reliability, and production of vibration. Therefore, the design techniques are required to find the tolerance band of design variables in order to minimize the cost and satisfy the performance in electric machine. Such a tolerance means uncertainty of design variables. Therefore, tolerance analysis is inevitably needed, considering the uncertainty of design variables. The uncertainty of design variables is considered by treating design variables as random parameters in electric machine.

The usual method for tolerance analysis is Monte Carlo Simulation. However, the major disadvantage of this method is that it requires a great number of computations to have an acceptable precision of statistically significant results. The number of samples will be very high, with a very large computational cost. This paper introduces Stochastic Response Surface Methodology (SRSM) to evaluate the statistical properties in the electric machine performance. The SRSM approximates the output function by using a polynomial fitting and samples the approximation to calculate specific statistical quantities of outputs [2]. These quantities of both input and output uncertainties are needed to define patterns of their variability and then are used to achieve the tolerance analysis in electric machines.

As an example, the tolerance analysis using SRSM is applied to design for a BLDC motor from the electric point of view. The aim of the design is to reduce the torque ripple of the BLDC motor based on conventional Response surface Methodology (RSM) and accomplish the tolerance analysis of design parameters to satisfy variation band of outputs.

II. CONCEPTS AND STATISTICAL FITTING METHOD

A. *Concept of Response Surface Methodology*

The RSM seeks to find the relationship between design variable and response through statistical fitting method. A polynomial approximation model is commonly used for a second-order fitted response and can be written as follow [3]:

$$u = \beta_0 + \sum_{j=1}^{k} \beta_j x_j + \sum_{j=1}^{k} \beta_{jj} x_j^2 + \sum_{i \neq j}^{k} \beta_{ij} x_i x_j + \varepsilon \tag{1}$$

where, β is regression coefficients, ε denotes the random error. The least squares method is used to estimate unknown coefficients. Matrix notations of the fitted coefficients and the fitted response model should be

$$\hat{\boldsymbol{\beta}} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{u} \tag{2}$$

$$\hat{\mathbf{u}} = \mathbf{X}\hat{\boldsymbol{\beta}} \tag{3}$$

B. Concept of Stochastic Response Surface Methodology

The SRSM can be illustrated as an extension of the deterministic Response Surface Methodology (RSM), and then the major difference being that in the former the input variables are random variables while in the latter the input variables are deterministic variables [2].

The first step in the implementation of the SRSM is to represent all input parameters in terms of random variables. Random variables with normal distributions, N(0,1), are frequently selected to represent input uncertainties because it is easy to deal with a mathematical function of these random variables. In this paper these random variables are mentioned to as standard random variables. When the random input variables are independent, the uncertainty in the i-th input of the model, x_i, is transposed properly as a function of the i-th standard random variables, ξ_i, by applying an appropriate transformation according to the following Table I.

In second step of the SRSM, a relationship of the uncertainty between the outputs and inputs is addressed by the series expansion of standard normal variables in terms of Hermite polynomials. Therefore, the output can be approximated by an expansion known as polynomial chaos expansion [2], [4]. The explain is as follows:

$$\begin{aligned} y = a_0 &+ \sum_{i1=1}^{n} a_{i1}\Gamma_1(\xi_{i1}) + \sum_{i1=1}^{n}\sum_{i2=1}^{i1} a_{i1i2}\Gamma_2(\xi_{i1},\xi_{i2}) \\ &+ \sum_{i1=1}^{n}\sum_{i2=1}^{i1}\sum_{i3=1}^{i2} a_{i1i2i3}\Gamma_3(\xi_{i1},\xi_{i2},\xi_{i3}) + \cdots \end{aligned} \tag{4}$$

where y is output of the model, a's are unknown coefficients to be estimated, $\Gamma_p(\xi_i)$ is Hermite polynomials of degree p, written as follows:

TABLE I. TRANSFORMATION FOR SOME OF THE COMMON DISTRIBUTIONS AS STANDARD RANDOM VARIABLE

Description pattern of input variables	**Transformation**
Normal (μ, σ)	$\mu + \sigma\xi$
Lognormal (μ, σ)	$e^{(\mu + \sigma\xi)}$
Gamma (a, b)	$ab(\xi\sqrt{(1/9a)} + 1 - (1/9a))^3$

* μ : mean value, σ : standard deviation, ξ : normal distribution (0,1).

$$\Gamma_p(\xi_{i_1}, \ldots, \xi_{i_p}) = (-1)^p e^{\frac{1}{2}\xi^T\xi} \frac{\partial^p}{\partial\xi_{i_1}\ldots\partial\xi_{i_p}} e^{-\frac{1}{2}\xi^T\xi} \tag{5}$$

where $\boldsymbol{\xi}$ is the vector of p standard random variables $\{\xi_{ik}\}_{ik=1}^{p}$, that are used for describing the uncertainty in the input. The Hermite polynomials on $\{\xi_i\}_{i=1}^{n}$.are random variables, because they are functions of the random variables. Furthermore, the Hermite polynomials defined on $\{\xi_i\}_{i=1}^{n}$ are orthogonal with respect to an inner product defined as the expectation of the product of two random variables.

$$E(\Gamma_p\Gamma_q) = 0 \ \text{ if } \ \Gamma_p \neq \Gamma_q \tag{6}$$

Therefore, a model output y with the uncertainty can be represented as a second order polynomial approximation and as follows:

$$y = a_0 + \sum_{i=1}^{n} a_i\xi_i + \sum_{i=1}^{n} a_{ii}(\xi_i^2 - 1) + \sum_{i\neq j}^{n} a_{ij}(\xi_i\xi_j) \tag{7}$$

where n is the number of standard random variables used to represent the uncertainty input in the model and the coefficients a is the unknown coefficients to be estimated.

The third step is to estimate parameters in the functional approximation of outputs. The unknown coefficients in the polynomial chaos expression can be estimated through regression method based on some sample points. And the following sections describe the statistical properties of the outputs as last step in SRSM.

III. BASIC STATISTICS FOR TOLERANCE ANALYSIS

Uncertainty of design variables can affect a performance of electric machines. Accordingly, it may cause variation in the performance. It is necessary for tolerance analysis of design variables to estimate a variation band of the output according to uncertainty of that. The variation band and uncertainty of design variables, with assuming the distribution of a normal distribution, is shown in Fig. 1. In this symmetrical distribution, the tolerance band of design variables is easy to quantify in terms of the percentage of the area that will occur between one, two and three standard deviation from the mean μ as follows [5]:

$$\Delta x = \pm n\sigma \quad (n = 1, 2, 3, \cdots) \tag{8}$$

Modeling variation of outputs according to tolerance of design variables is made by the SRSM. From a set of N samples, the basic moments of the distribution of an output y_i can be calculated as follows:

$$\mu_{y_i} = E\{y_i\} = \frac{1}{N}\sum_{j=1}^{N} y_{i,j} \tag{9}$$

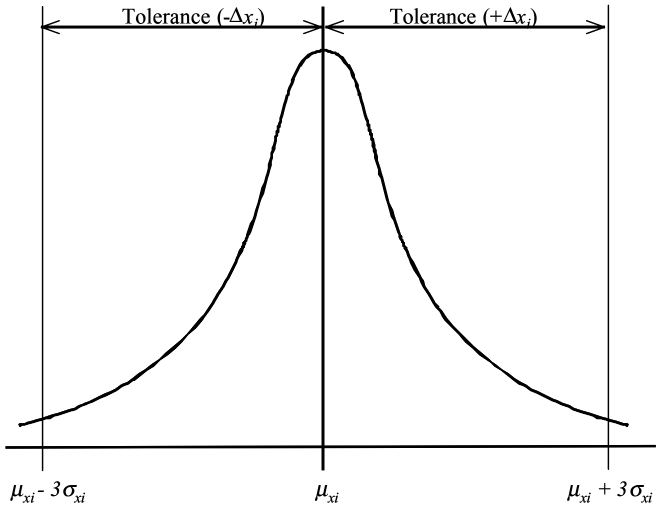


Fig. 1. Tolrerance band of design variables

$$\sigma_{y_i}^2 = \mathrm{E}\{(y_i - \mu_{y_i})^2\} = \frac{1}{N-1}\sum_{j=1}^{N}(y_{i,j} - \mu_{y_i})^2 \tag{10}$$

$$\sigma_{y_i} = \sqrt{\sigma_{y_i}^2} \tag{11}$$

where, μ_{yi} is a mean, $\sigma_{y\,i}^2$ is a variance and σ_{yi} is a standard deviation, respectively.

IV. APPLICATION FOR TOLERANCE ANALYSIS

Recently, the demand for BLDC motors is expanding rapidly, and better good quality is required in some of applications. So, BLDC motors have to be designed in order to design specifications. The torque ripple of BLDC motors arises from the interaction between its rotor magnet and slotted stator. It exerts a bad influence on the motor performance. Therefore, this paper illustrates that the torque performances of the BLDC motor is applied to a reduction of the torque ripple and tolerance analysis of controller factors, which is using deterministic Response Surface Methodology and Stochastic Response Surface Methodology, respectively.

A. Field Computation Framework

The magnet field within the motor is computed using the two-dimensional finite element method (2-D FEM). The analysis domain comprises a sixth model of the whole motor and periodic conditions are used as boundary conditions of analysis model. The Maxwell stress tensor is used for a resultant forces and torque calculations. Thus, a torque ripple is defined as follows:

$$\mathrm{T_{Ripple}} = \frac{\mathrm{T_{Peaktopeak}}}{2\,\mathrm{T_{Mean}}} \tag{12}$$

where, $\mathrm{T_{Peak\ to\ peak}}$ is difference between the maximum and the minimum of the running torque and $\mathrm{T_{Mean}}$ is a mean value of a running toque. Moreover the rotor rotation is simulated by moving-line technique without the regeneration of meshes of the analysis model.

B. Model and Define Design Variables Framework

The applied machine is a BLDC motor used for an electric power steering of a vehicle. The stator has 18 slots and the rotor is built of 12 title of radial magnetic, bonded NdFeB magnet. The motor applied for an electric power steering is requiring a low level of the torque ripple for comfortable steering of a vehicle. The motor geometries can be defined by 9 parameters as shown on Table II and Fig. 2.

As 9 parameters define the shape of the motor, in this case the simulation time is long due to the number of the required experiments, although without taking into account the interactions of high order between parameters. Therefore, it is necessary for an investigation process of the significant parameters. Fractional factorial designs are well adapted to this problem and furthermore only 12 simulations with using taguchi's orthogonal array L_{12} [6]-[8]. Applying the sum of squares versus various parameters makes possible to select the significant parameters and Fig. 3 shows the importance of each parameter and three controller factors are selected as x_2, x_4, x_8.

C. Optimization Framework

The general formation of a conventional optimization is expressed as following.

TABLE II. PARAMETER DESCRIPTION

Parameters	DESCRIPTION	Initial Value
x_1	Dead zone (edeg.)	18.0
x_2	Skew angle (mdeg.)	0
x_3	Stator yoke thickness (mm)	4
x_4	Tooth width (mm)	4
x_5	Tooth shield angle (mdeg.)	98
x_6	Slot open width (mm)	2
x_7	Tooth shield height (mm)	1
x_8	Aiir-gap (mm)	0.8
x_9	Slot fillet radius (mm)	1

* edeg : electrical degree , mdeg : mechanical degree

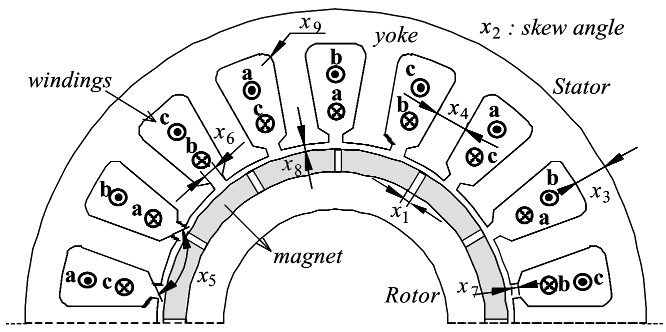


Fig. 2. Analysis model and design variables

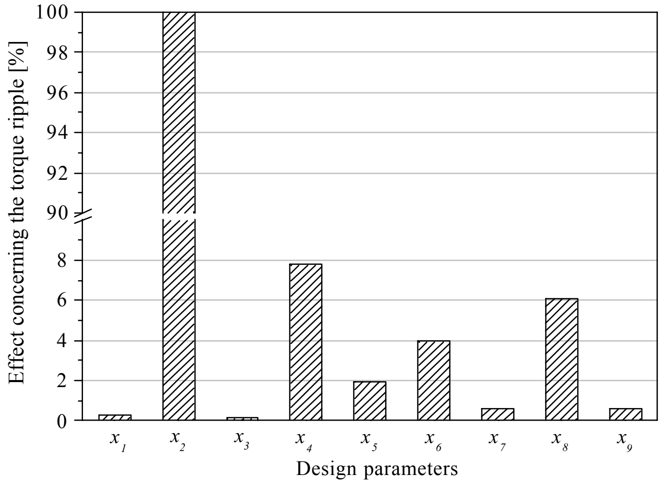


Fig. 3. Effects of design variables regarding the torque ripple

$$\text{Minimize: } f(x_1, x_2, \cdots, x_k) \tag{13}$$

$$\text{Subject to: } g_i(x_1, x_2, \cdots, x_k) \leq 0, \qquad i = 1, 2, \cdots, m \tag{14}$$

$$x_{iL} \leq x_i \leq x_{iU} \qquad i = 1, 2, \cdots, k \tag{15}$$

where, $f(x_1, x_2, \cdots, x_k)$ is the objective function, $g_i(x_1, x_2, \cdots, x_k)$ is the constraint functions with the dimension of m, x_{iL} and x_{iU} is lower and upper bounds of design variables x_i and k is the total number of design variables respectively.

The goal of the optimization is to reduce the torque ripple and to satisfy the running torque. An analytical model built from the deterministic Response surface Method (RSM) can be used as either objective functions or constraint functions in an optimization procedure and the sequential quadratic programming method [9] has been used to solve this work.

D. Tolerance Analysis Framework

In order to consider uncertainty of design variables, the mean (μ_{xi}) and standard deviation (σ_{xi}) of them are obtained from (8) and numerical results of the optimization described following section. The tolerances of design variables are regarded as 3σ and their distribution are assumed as normal distributions, and the fundamental moments of design variables are calculated with assuming uncertainty, which is 10(%), 5(%) and 1(%), respectively. Table III presents the moments of design variables.

TABLE III. MOMENTS ACCORDING TO TOLERANCES OF DESIGN VARIABLES

Section	μ_{xi}	σ_{xi} at Δx_{xi} = ±10 %	σ_{xi} at Δx_{xi} = ±5 %	σ_{xi} at Δx_{xi} = ±1 %
Skew angle (x_2)	10.2 (*deg.*)	0.340	0.170	0.034
Tooth width (x_4)	4.23 (*mm*)	0.141	0.071	0.014
Air-gap (x_8)	0.76 (*mm*)	0.025	0.013	0.003

V. NUMERICAL RESULTS AND DISCUSSION

This paper present the tolerance analysis after the optimization design based on the statistical fitting method. For applying tolerance analysis on the BLDC motor, firstly, taguchi's orthogonal design is used to select the three main factors of the nine design parameters as described above section and the optimization design to reduce the torque ripple is executed by using the second-degree fitted model of the torque performance on the BLDC motor then the tolerance analysis is accomplished by SRSM. At this work, the optimization results are used as the expectation value of the design variables. A schematic depiction Fig. 4 describes the tolerance analysis procedure of the applied BLDC motor.

A. Results for Numerical Optimization

The second-order fitted model of the torque ripple is used as the objection function and that of the running torque is used as the constraint function, respectively. For building the second-order fitted model, besides, the central composite design (CCD) is used in this paper. CCD is frequently used for fitting second-order response model and CCD involving three factors is required to conduct 15 experiments. Axial points on the axis of three design variables at a distance from the design center choose 1.682 for a rotatable experiment design [10]. The levels of three design variables are shown in Table IV and the toque performance is simulated using 2-D FEM in each trial. The two regression models are defined as objective function and the constraints as follows:

Minimize:

$$\begin{aligned} f(x) &= \hat{u}_{\text{Torque ripple}} \quad (\%) \\ &= 40.184 - 10.485x_2 + 26.975x_4 + 2.676x_8 \\ &\quad + 0.482x_2^2 + 0.735x_4^2 + 0.349x_8^2 \\ &\quad + 0.255x_2x_4 + 0.099x_2x_8 - 8.599x_4x_8 \end{aligned} \tag{16}$$

Subject to:

$$\begin{aligned} g_1(x) &= \hat{u}_{\text{Running torque}} \quad (kgf \cdot cm) \\ &= -29.462 + 0.291x_2 + 4.109x_4 + 33.719x_8 \\ &\quad -0.016x_2^2 - 1.425x_4^2 - 3.612x_8^2 - 0.001x_2x_4 \\ &\quad -3.492x_4x_8 \geq 41 \end{aligned} \tag{17}$$

and the design space is chosen as follows:

$$7.32 \leq x_2 \leq 10.68,\ 3.66 \leq x_4 \leq 4.34,\ 0.53 \leq x_8 \leq 0.87 \tag{18}$$

TABLE IV. LABEL OF THE CONTROLLABLE PARAMETERS

Controllable parameters		**The label of the parameters**				
		-1.682	**-1**	**0**	**1**	**1.682**
x_2	**Skew angle (mdeg.)**	7.32	8.0	9	10.0	10.68
x_4	**Tooth width (*mm*)**	3.66	3.8	4.0	4.2	4.34
x_8	**Air-gap (*mm*)**	0.53	0.6	0.7	0.8	0.87

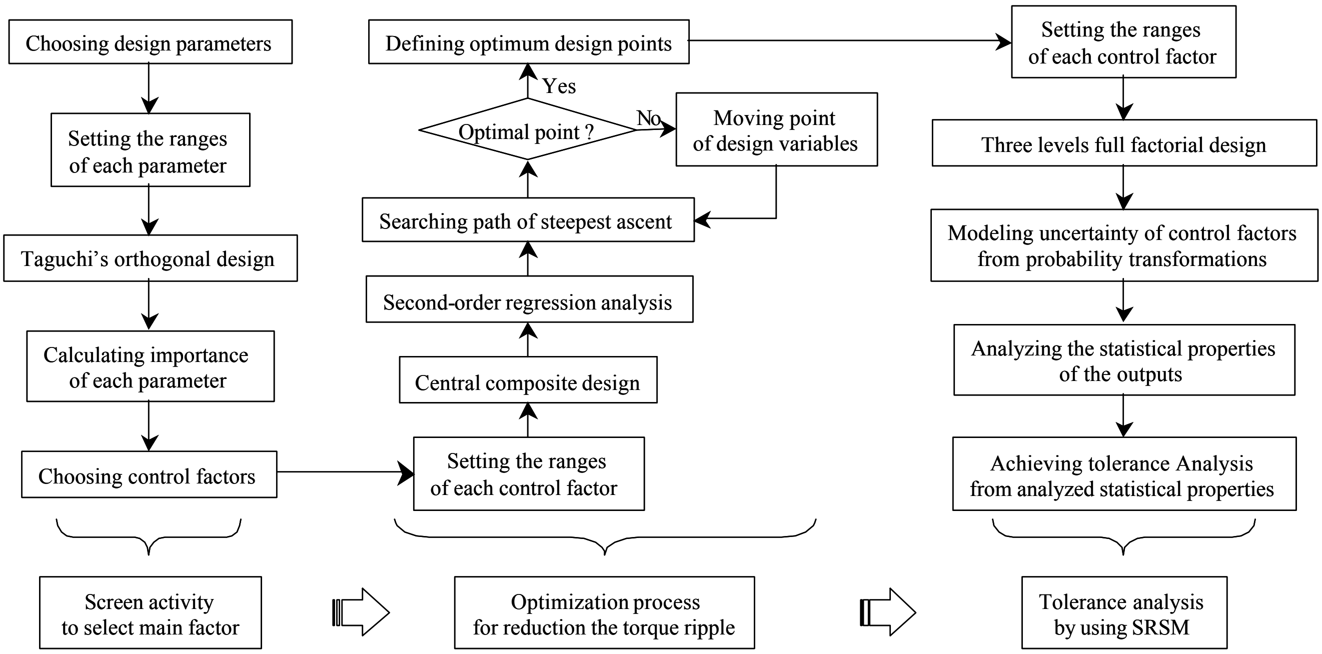


Fig. 4. Procedure of the tolerance analysis in this work

The aim of the optimization can be accomplished through substituting (16)-(18) in (13)-(15), one by one. The result of this optimization design is shown in Table V the predicted response surfaces versus design variables are shown in Fig. 5 and optimum point is illustrated on these response surfaces, respectively.

The polynomial chaos expression of the torque characteristic is used to analyze the tolerance and can be written as follows;

Polynomial chaos expression of the torque ripple:

$$y_{\text{Torque ripple}} = 0.579 - 0.011x_2 - 0.144x_4 + 0.015x_8 + 0.056(x_2^2 - 1) + 0.001(x_4^2 - 1) + 0.007(x_8^2 - 1) + 0.002x_2x_4 + 0.005x_2x_8 - 0.031x_4x_8 \quad (19)$$

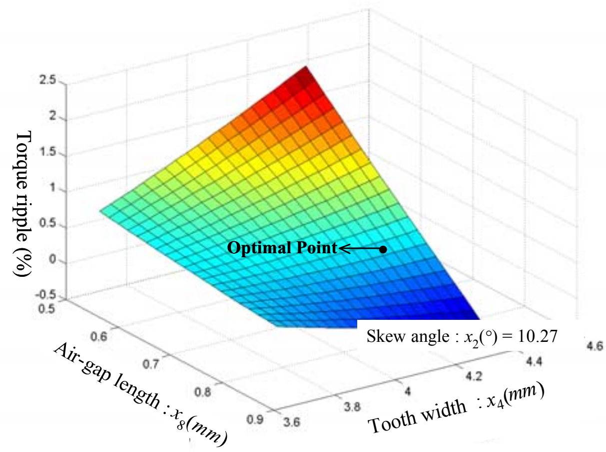
TABLE V. RESULTS OF THE NUMERICAL OPTIMIZATION

Design variables	Initial Model		
	Size	**Running torque**	**Torque ripple**
Skew angle (x_2)	0 (*deg.*)	41.3 (*kgf.cm*)	2.24 (%)
Tooth width (x_4)	4.0 (*mm*)		
Air-gap (x_8)	0.8 (*mm*)		
Design variables	**Optimal Model**		
	Size	**Running torque**	**Torque ripple**
Skew angle (x_2)	10.2 (*deg.*)	41.4 (*kgf.cm*)	0.54 (%)
Tooth width (x_4)	4.23 (*mm*)		
Air-gap (x_8)	0.76 (*mm*)		

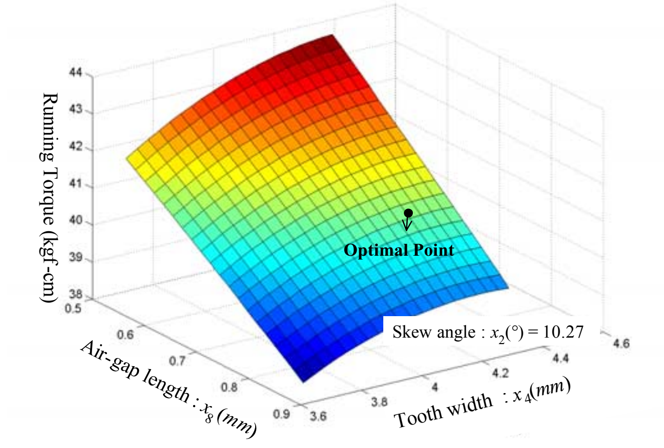
Polynomial chaos expression of the running torque:

$$y_{\text{Running Torque}} = 40.868 - 0.011x_2 - 0.325x_4 + 0.072x_8 - 0.002(x_2^2 - 1) - 0.001(x_4^2 - 1) - 0.072(x_8^2 - 1) - 0.013\,x_4x_8 \quad (20)$$

For building the second-order polynomial chaos expression, besides, the full factorial design (FFD) is used in this paper. FFD with three factors is required to conduct 27 experiments. The levels of three design variables are shown in Table VI. The tolerance analysis can be accomplished through substituting (19), (20) in (9)-(11). Also, the distributions of design variables are assumed as normal random variable. The statistical characteristic of the output is computed by using the random numbers generate with standard normal distribution.



(a) The response surface regarding the torque ripple



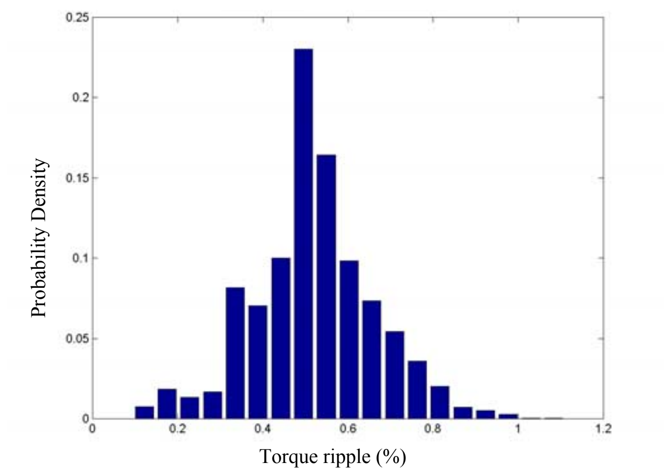
(b) The response surface regarding the running torque

Fig. 5. The predicted response surface

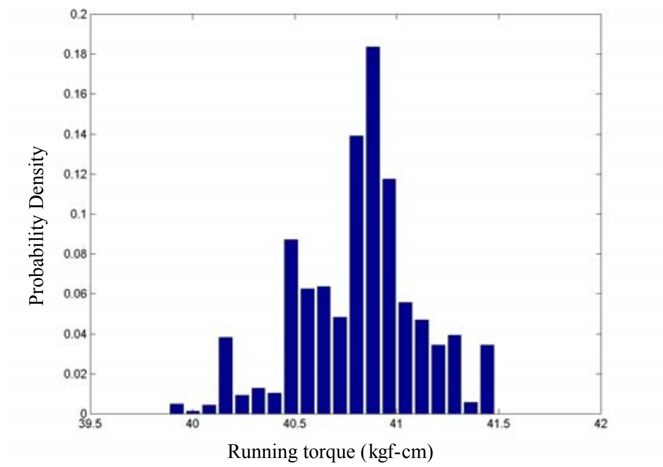
In this paragraph, the results of tolerance analysis show the practical application of the SRSM using the result from the optimization design. The statistic used in tolerance analysis will typically be produced from proper sample data. In this paper, data from populations of outputs is used as the number of 8000 samples in all cases, if the manufacturing process is running at the design variable tolerance of 10 (%) based on the three-sigma level. The variation of the outputs is distributed as shown in Fig. 6.

TABLE VI. THE LABEL OF THE DESIGN PARAMETER

Controllable parameters		The label of the parameters		
		-1.74	**0**	**1.74**
x_2	**Skew angle (mdeg.)**	9.6	10.2	10.7
x_4	**Tooth width (mm)**	3.99	4.23	4.47
x_8	**Air-gap (mm)**	0.72	0.76	0.8



(a) Distribution concerning the torque ripple



(b) Distribution concerning the running torque

Fig. 6. Distribution of outputs at the design variable tolerance of 10 (%)

In order to reduce scatter of the outputs from their population, design variables need to run at a tighter tolerance than 10 (%), such as 5(%). The scatter of population of outputs at the design variable tolerance of 5(%) gravitates toward the means of the outputs. However, if all design variables are regulated with tight tolerance, the manufacturing cost is increased. The sensitivity, which is obtained from variance analysis, of each effect for three design variables, is shown in Fig. 7. From the results of variance analysis, the air-gap (x_8) needs to be controlled with a tighter tolerance than the others. The results according to running tolerances are compared with each case and then they are shown in Fig 8. These results show, when the air-gap (x_8) is controlled with a proper tight tolerance and the tolerance of the others are is regulated with loose tolerance, the scatter of population of outputs extremely centralizes in the means of the outputs, compared that all design variables are regulated with tight tolerance.

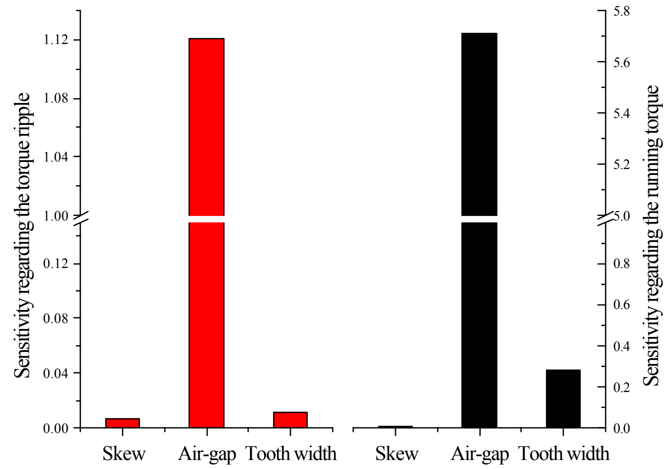
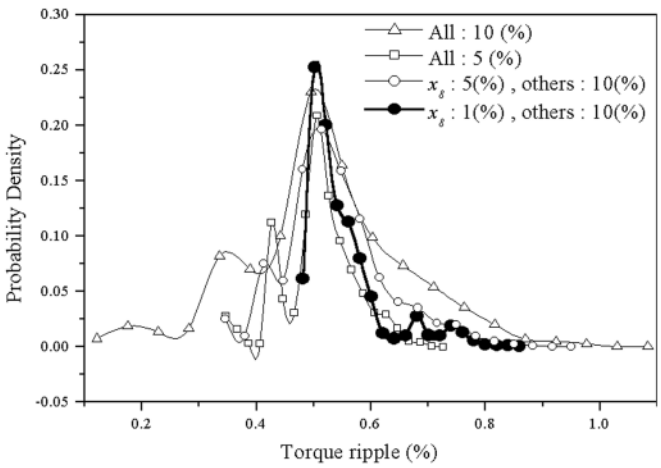
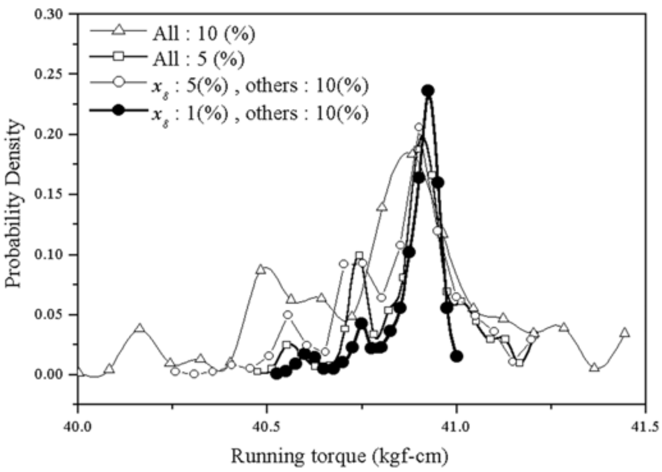


Fig. 7. Sensitivities of each design variable



(a) The predicted distribution concerning the torque ripple



(b) The predicted distribution concerning the running torque

Fig. 8. Compared with predicted distributions of each cases

VI. CONCLUSION

The application considered in this paper is on the tolerance analysis of the torque characteristics of the BLDC Motor from the electric point of view. The tolerance analysis is accomplished by the SRSM, which is used to estimate statistically significant results of the torque characteristics of the BLDC motor. In most cases of manufacturing electric machines, manufacturing tolerance inevitably occur because of the necessity for a manufacture process and such tolerances certainly have an influence on the machine's performance. Therefore, tolerance analysis is very important in the electric machine industry for improving product robustness and reducing the cost. If it is possible to allow for reasonable tolerance in each design variable in the design stage, this approach will provide a great potential for the cost reduction without losing performance.

REFERENCES

[1] Y.K. Kim, J.P. Hong, J Hur, "Robust Optimization of Electric Machine using Stochastic Finite Element Method," *JOURNAL OF ELECTRICAL ENGINEERING*, vol. 1/2001, No.1, pp. 69-72, January 2001.

[2] S. S. Isukapalli, et all, "Stochastic Response Surface Methods (SRSMs) for Uncertainty Propagation: Application to Environmental and Biological Systems. Risk Analysis, 18(3): 351-363, 1998.

[3] Y K. Kim, Y S Jo, J P Hong, J Lee, "Approach to the Shape Optimization of Racetrack Type High Temperature Superconducting Magnet Using Response Surface Methodology," *Cryogenics*, vol. 41/1, pp.39-47, 2001.

[4] R. G. Ghanem and P. D. Spanos, Stochastic Finite Elements: A Spectral Approach, Springer-Verlag, New York, 1991.

[5] C. M. Creveling, *TOLERANCE DESIGN A HANDBOOK FOR DEVELOPING OPTIMAL SPECIFICATIONS*, Addison-Wesley, 1995.

[6] M. S. Phadke, *QUality ENGINEERING USING ROBUST DESIGN*, Prentice hall, 1989

[7] S. X. Chen, T. S. Low, B. Bruhl, "The Robust Design Approach for Reducing Cogging Torque in Permanent Magnet Motors," *IEEE Trans. Magn.*, vol. 34, No. 4, pp. 2135-2137, Jul. 1998.

[8] F. Gillon, P. Brochet, "Screening and Response Surface Method Applied to the Numerical Optimization of Electromagnetic Devices," *IEEE Trans. Magn.*, vol. 36, No. 4, pp. 1163-1167, Jul. 2000.

[9] J. S. Arora, *Introducton to Optimum Design*, McGraw Hill, 1989.

[10] R. H. Myers and D. C. Montgomery, *Response Surface Methodology*, John wiley & Sons, INC., 1995.