

# Characteristic Analysis of 5-phase Hybrid Stepping Motor Considering the Saturation of Both Teeth and Poles

Ki-Chae Lim

Jung-Pyo Hong, *Member, IEEE*

Gyu-Tak Kim

Department of Electrical Engineering, Changwon National University,  
#9 Sarim-dong Changwon city Kyungnam, Korea, 641-773  
E-mail:kclim@cosmos.changwon.ac.kr/+82-551-279-7519

**Abstract**—The paper presents the characteristic analysis methodology for 5-phase hybrid stepping motors used for electro-magnetic incremental-motion actuators. The approach is based on the use of equivalent magnetic circuit taking into account the localized saturation throughout the hybrid stepping motor. Lumped circuit parameters are calculated using the non-linear finite element analysis for a given geometry and the neural network is used to map a change of parameters and predicts their approximation. Therefore, the proposed method efficiently improves the accuracy of analysis by using the parameter characterizing localised saturation effects and reduces the computational time by using the neural network. An improved circuit model of 5-phase hybrid stepping motors is presented and its application is provided to demonstrate the effectiveness of the proposed method.

**Index term** : 5-phase hybrid stepping motor, saturation of both teeth and poles, finite element(FE) analysis.

## I. INTRODUCTION

Although many advanced motors of recent are developed for servo systems, stepping motors are still good solution for many industrial and office automatic applications as they can provide accurate position and speed control in open-loop system. Among various types of the stepping motors, hybrid stepping motors having a permanent magnet on the rotor and doubly-salient structure on the stator and the rotor are the most commonly used in industry as having higher torque and resolution advantages over other types of the stepping motors[1].

Especially, 5-phase hybrid stepping motors have better performance than 2-phase hybrid motors due to the fact that they produce a low torque ripple and have a good low-speed characteristics with high-torque.

The accurate prediction to the performance of electric machines has been a major concern for researchers and it has been dramatically improved by using the numerical method to obtain more precise solution for problem. The finite element method(FEM) has been one of the most powerful and widely used technique in machine design and analysis. However, it has a disadvantage of excessive demand for computational time and resources. Especially, the application to analysis for hybrid stepping motors is more inefficient task and may generate unreliable results because of small airgap and 3-D nature of motor. Therefore, the equivalent magnetic circuit method with lumped circuit parameters has been widely applied to predict the performance of the hybrid stepping motor.

But, it is difficult to obtain clear equivalent circuit of hybrid stepping motors than that of other type motors because they are actually a 3-D device of which main flux takes both radial and axial paths. And, its circuit parameters cannot be easily calculated because they are designed to operate under highly saturated conditions to achieve a competitive torque per the unit volume[1]. The

equivalent circuit method is not always accurate enough but flexible and time-saving for the changes of circuit parameters and its accuracy can be archived by improving the precision of lumped parameters.

To analyze the hybrid stepping motor accurately, two different methods are used; the equivalent magnetic circuit and the FE analysis.

The lumped parameters of each flux path are required to apply the equivalent magnetic circuit. Especially, the tooth/airgap permeance represented by the permeance of both the stator/rotor teeth and airgap region plays a important role in torque/displacement characteristics. In addition, the reluctance of each stator pole must be considered due to the saturation effect, as the exciting current increases over the rated current.

To calculate the airgap permeance, the permeance based analytic method has been used; which calculates the space distribution of the permeance by using quasi-flux tubes determined by straight lines and semicircular segments drawn to maximize the permeance of each flux path[2,3].

It, however, cannot consider the saturation of iron core and the geometric complexity which significantly affect the machine performance. As alternatives, FEM is introduced to obtain the permeance data reliable enough[4]. And, using the neural network, we calculated the total distribution of permeance considering the saturated characteristics.

The equivalent circuit coupled with FEM is solved by the net matrix which describes the relationship between MMF sources and lumped parameters. As results, the virtual work technique is applied to compute the torque/displacement characteristics.

Fig. 1 shows the basic construction of 5-phase hybrid stepping motors, which is nearly identical to 2-phase hybrid stepping motors.

## II. EQUIVALENT MAGNETIC CIRCUIT MODEL

Fig. 2 shows a cross-sectional view of a widely used 5-phase hybrid stepping motor having 10 stator poles with 4 teeth and 50 teeth on rotor and Fig. 3 is a side view and displays the flux paths of the motor.

From Fig. 2 and Fig. 3, the equivalent magnetic model of the 5-phase hybrid stepping motor can be derived from two orthogonal fields; a radial field produced by stator phase windings and an axial field produced by the permanent magnet on the rotor[1].

Fig. 4 describes the equivalent magnetic circuit model for only 5 stator poles due to the symmetrical construction of the motor. The permeances between A-phase pole on the stator and N- and S-pole rotors are defined  $P_{AN}$  and  $P_{AS}$ , respectively. And the other symbols expressed as the permeance form can be understood easily in the same manner. As the rotor and stator are normally made of a

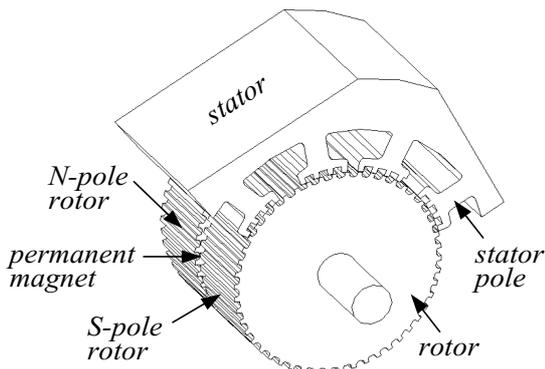


Fig. 1. Basic structure of hybrid stepping motors

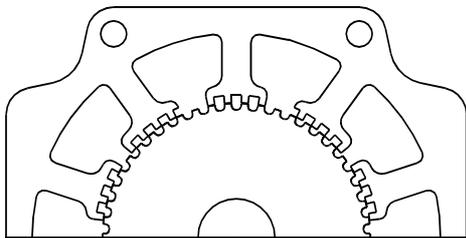


Fig. 2. A cross-sectional view for hybrid stepping motor

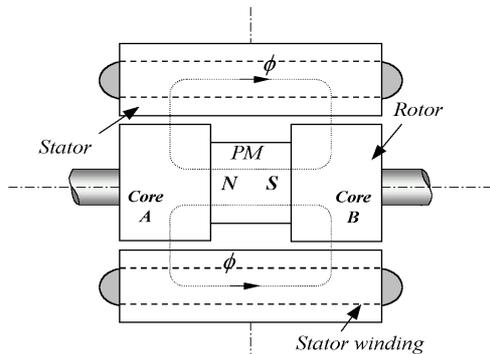


Fig. 3. Flux paths of the hybrid stepping motor

laminated silicon steel, they can be considered as magnetic resistors (reluctances), expressed by  $R_r$  and  $R_s$ , respectively, for the axial flux path due to the permanent magnet.

The permanent magnet can be modeled as an ideal magneto-motive force (MMF) having an internal reluctance expressed by  $R_m$ . And, each MMF by the phase winding producing the radial flux is defined as symbol  $F$  including subscript A, B, C, D, and E representing the current phases, which is a product between the number of turns of stator winding and the excited current of each phase.

### III. CALCULATION OF PERMEANCE

The equivalent circuit method is a simple and flexible yet still an accurate technique for predicting the machine

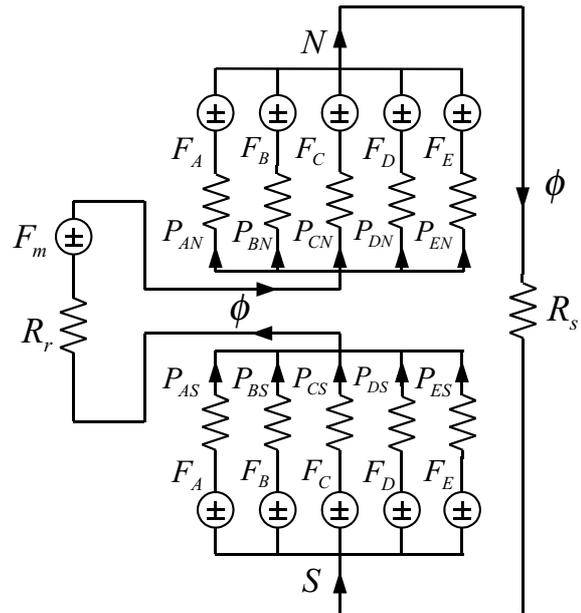


Fig. 4. Equivalent magnetic circuit

performance because the circuit parameters mostly include non-linear magnetic property and detailed geometry. While the FEM has been the most prevalent technique not only to estimate the machine performance but also to calculate the machine parameters because of its high accuracy.

Torque production of hybrid stepping motors is vitally related to the variations in the airgap permeance with respect to the incremental motion of the rotor. The airgap permeance depends on the shape of the teeth, the size of the airgap, the level of the magnetic saturation, and the relative position of the rotor. Thus, the permeance representing the circuit parameters is calculated by using the FEM for a given condition.

The use of FEM in calculation of the circuit parameters provides a significant advance in the degree of accuracy with which the equivalent circuit can be modeled at the price of excessive computing time.

In this paper, the FEM formulated from the magnetic scalar potential is introduced to calculate the permeance.

Equation (1) is the governing equation for the magnetic scalar fields.

$$\nabla^2 \phi = 0 \quad (1)$$

where  $\phi$  is a scalar magnetic potential.

By applying the FEM for the single tooth pitch of the tooth/airgap region, the airgap permeance  $P_g(\theta_e)$  can be calculated by both the magnetic stored energy  $W_s$  and the given MMF  $F_g$  in (2).

$$W_s = \frac{1}{2} F_g^2 P_g(\theta_e) \quad (2)$$

where,

$\theta_e$ : Electrical angle, which is displacement from 0 to  $2\pi$ , corresponds to one rotor tooth pitch and is

derived by a product of mechanical angle  $\theta_m$  and the number of teeth of the rotor,  $N_r$ .

Therefore, the airgap permeance can be expressed by the following Fourier series form for the N-poles of the 5-phase hybrid stepping motor[2].

$$\begin{aligned} P_{gAN} &= P_0 + \sum_{i=1}^{\infty} P_i \cos(\theta_e) \\ P_{gBN} &= P_0 + \sum_{i=1}^{\infty} P_i \cos(\theta_e + \frac{4}{5} \pi) \\ P_{gCN} &= P_0 + \sum_{i=1}^{\infty} P_i \cos(\theta_e + \frac{8}{5} \pi) \\ P_{gDN} &= P_0 + \sum_{i=1}^{\infty} P_i \cos(\theta_e + \frac{12}{5} \pi) \\ P_{gEN} &= P_0 + \sum_{i=1}^{\infty} P_i \cos(\theta_e + \frac{16}{5} \pi) \end{aligned} \quad (3)$$

where,

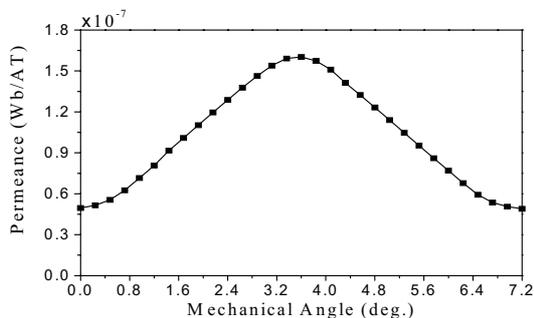
$P_0$  and  $P_i$  : Permeance coefficients

$i$  : Order of harmonics

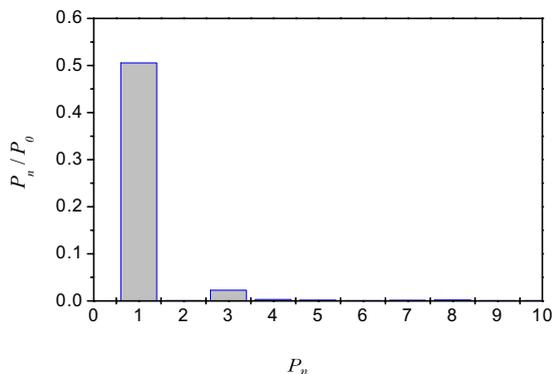
From (3), the airgap permeance for the S-poles is clearly written by replacing  $\theta_e$  by  $\theta_e + \pi$  due to the fact each N- and S-pole is offset by half of rotor tooth pitch corresponding to electrical angle  $\pi$ .

The permeance coefficients actually represent the space harmonic distribution for the permeance curve with respect to the rotor position obtained from the FE analysis. And the permeance variations expressed by (3) are coupled with the equivalent circuit to estimate the performance of the motor.

Fig. 5 shows an airgap permeance curve and its space harmonic distribution on the assumption that the relative



(a) Airgap permeance curve



(b) Space harmonic distribution

Fig. 5. Airgap permeance characteristic

permeability of the iron core is infinite and the leakage flux in the axial direction can be neglected. The result of Fig. 5 is taken from the airgap model and not affected by the change of the applied MMF across the airgap.

#### IV. CHARACTERISTIC ANALYSIS

The magnetic flux in each branch of the magnetic circuit is determined by solving the equivalent magnetic circuit shown in Fig. 4 when the circuit parameters are calculated.

By the principle of virtual work, the torque developed by the rotor is found as the derivative of the magnetic stored energy in the analysis model,  $W_s$ , with respect to the rotor position at constant flux  $\Phi$ .

$$T = \frac{\partial W_s}{\partial \theta_m} \quad (4)$$

Since the flux is kept constant as the change of rotating angle, the corresponding torque expression can be obtained as (5)[5].

$$T = -\frac{1}{2} N_r F_a^2 \frac{\partial P(\theta_e)}{\partial \theta_e} \quad (5)$$

where,  $T$  is the static torque,  $F_a$  is the applied MMF, and  $P$  is the permeance.

Therefore, the static torque with respect to each rotating angle can be calculated by solving the equivalent magnetic circuit with its circuit parameters obtained by the FEM.

The permanent magnet type motors generally have a cogging torque even in the absence of source current because of the flux from the permanent magnet. And the torque formula is identical to (4).

To solve the equivalent magnetic circuit, it is important to predict the accurate permeance value, especially, the permeances between the stator pole and the rotor described by symbol  $P$  in the equivalent circuit.

The static torque neglecting the non-linearity and reluctance of the iron core can be easily obtained by using the permeance between the stator pole and rotor which has only a part of the airgap permeance shown in Fig. 5. In this case, the holding torque is estimated about 1.5 or 2 times greater than experimental result according to a different size of the 5-phase hybrid stepping motor.

The hybrid stepping motors, however, are designed to operate under saturated conditions, therefore, it requires the non-linear analysis method in the equivalent circuit.

Generally, the saturation in the hybrid stepping motor is chiefly developed by the stator/rotor teeth. For the purpose of taking the saturation effect of the stator/rotor teeth into account, the tooth/airgap model shown in Fig. 6, is treated in FE analysis. The tooth/airgap model is presented to calculate the permeance of the stator/rotor tooth and airgap and also includes the permeance of a part of both the stator pole and the rotor to think about the influence of the tooth saturation effect on the stator pole and the rotor. As shown in Fig. 6, the permeance between the stator pole and rotor of each phase is replaced by the permeance calculated by the FE analysis when the reluctance of the stator pole and the rotor is neglected.

Fig. 7 shows the FEM result for the tooth/airgap model

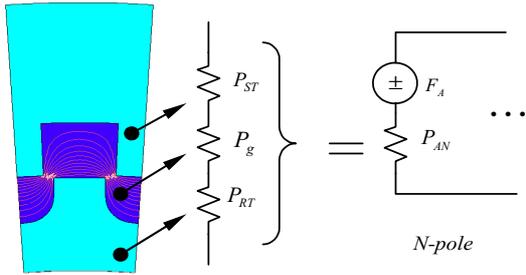


Fig. 6. Tooth/airgap permeance model

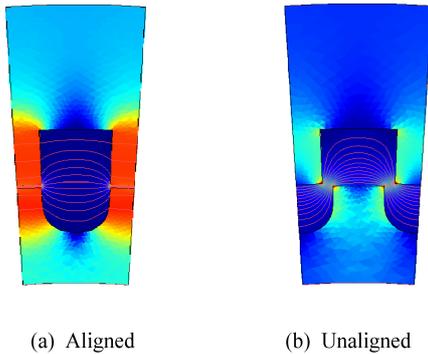


Fig. 7. FE analysis results for tooth/airgap model

when the stator/rotor teeth are aligned and unaligned with each other.

In case of considering the saturation of the stator/rotor teeth, the equivalent circuit cannot be solved at a time because its permeances are the function of the applied MMF drop which is unknown at the start of the problem and represents a degree of the saturation. Therefore, the circuit solution must be determined by an iterative process and keep correcting the permeance by the obtained MMF until it is consistent with the circuit solution.

Fig. 8 illustrates the whole process to treat the non-linear parameters in the equivalent circuit.

Firstly, the lumped parameters are initialized and the permeance of the tooth/airgap model is calculated by using FE analysis in which a temporary determined MMF drop across the permeance is used due to an initial permeance unknown. Secondly, the net flux is obtained by solving the equivalent circuit using the determined parameters and the MMF drop can be re-calculated using the net flux.

Finally, each non-linear permeance is corrected by the obtained MMF until it converges to the desired tolerance.

Although the permeance data should be re-calculated through the FE analysis when the new local MMFs are determined by solving the equivalent magnetic circuit, it is computationally time expensive. Consequently, the parameter approximation techniques are needed to reduce the computational time in predicting the non-linear permeance while maintaining a good accuracy of parameter estimation.

Fig. 9 shows the permeance distribution of the tooth/airgap model with respect to both the mechanical angle between the rotor and stator tooth and the MMF drop across the permeance and the permeance coefficients are taken from each permeance distribution for a given MMF

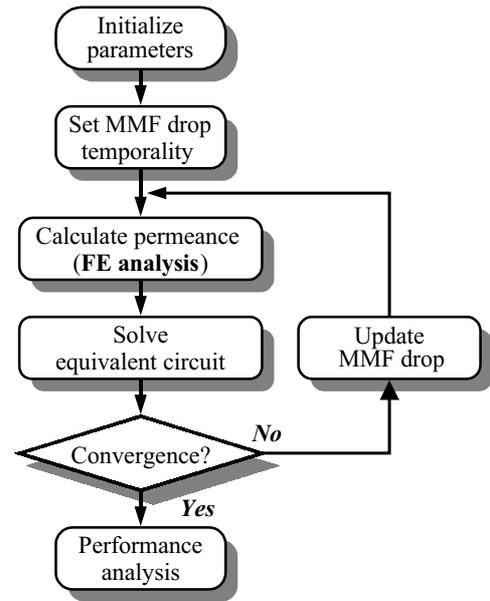


Fig. 8. Flow chart of performance analysis

drop.

When the hybrid stepping motor is used in over-rated current region, the saturation on stator poles cannot be ignored, especially, in condition that fluxes produced by both permanent magnet and exciting current are added on the stator pole. The saturation on the stator pole can be treated in the equivalent circuit by using its geometric dimension and the non-linear permeability computed by B-H characteristic curve.

## V. Neural Network Model

Artificial Neural networks are composed of simple elements called neuron operating in parallel. As in nature, the neural network is implemented largely by connection between neurons and capable of learning and storing complex information in various fields of application[6].

Fig. 10 shows a multi-layer feed-forward neural network which is generally composed of three different layers. The back-propagation is a very popular network model for most multi-layer feed-forward network and appropriate for the function approximation. In general, the neural network should be trained to create reasonable answers by learning algorithms based on other optimization techniques.

Fig. 11 illustrates the training process of the neural network which is employed to map the relations between MMF drop and rotor position as the input and magnetic circuit parameters as the output. The training process is achieved by using the well-known Levenberg-Marquardt algorithm which reportedly has the fastest convergence[6].

The number of neuron in the input and output layer is determined by the number of the input and output variables but the choice of the number of the hidden layers and neurons is not clearly determined but somewhat experimental work. In this application, the neural network has 4 layers with a 2-10-10-1 structure in which two hidden layers are used.

A series of different permeance data with respect to both MMF drop and rotor position in the feasible region

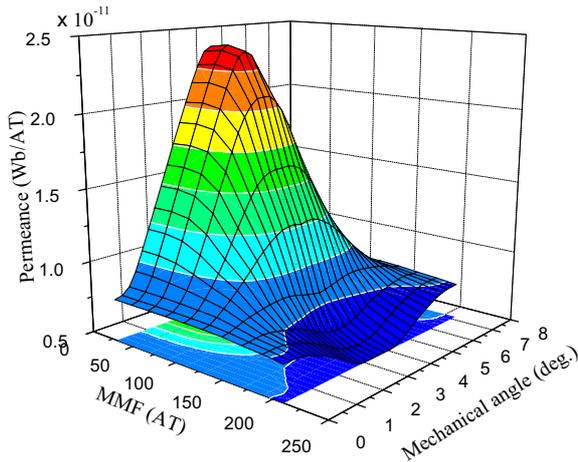


Fig. 9. Tooth/airgap permeance characteristics

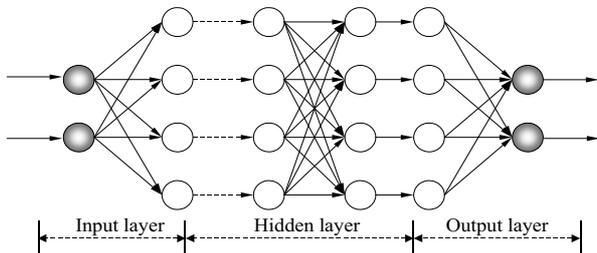


Fig. 10. Multi-layer feed-forward neural network

are calculated using FE analysis and used to train the neural network estimator. The permeance data to create the properly-trained neuron network are already shown in Fig. 9.

Fig. 12 shows the improved process of the performance analysis of 5-phase hybrid stepping motors in which the neural network is put into performance analysis process.

When training properly, the neural network has the ability to provide the output results of a given input very fast and estimates accurately the output results for an unknown input value in the trained region.

## VII. SIMULATION AND EXPERIMENT

The brief specifications of test motor, which has been widely used in industry application, is given in Table I.

Fig. 13 shows the permeance distribution calculated by the trained neural network and the FEM. They show a good correlation, however, the difference is still exist and slightly increased around an unaligned tooth position which represents zero mechanical angle. Fig. 13-(a) describes the direct comparison of the FEM results as the desired data and the neural network results as the estimated data and Fig. 13-(b) shows the calculation results at the center of mechanical angle value in Fig. 13-(a). And the unknown values according to MMF drop are successfully estimated.

For a given motor specifications, the solution time for the calculation of the equivalent circuit parameters by using the properly-trained neural network is by far less than the calculation time required by the FEM solution.

Therefore, the equivalent circuit method combined directly with the FEM can rarely used in real application.

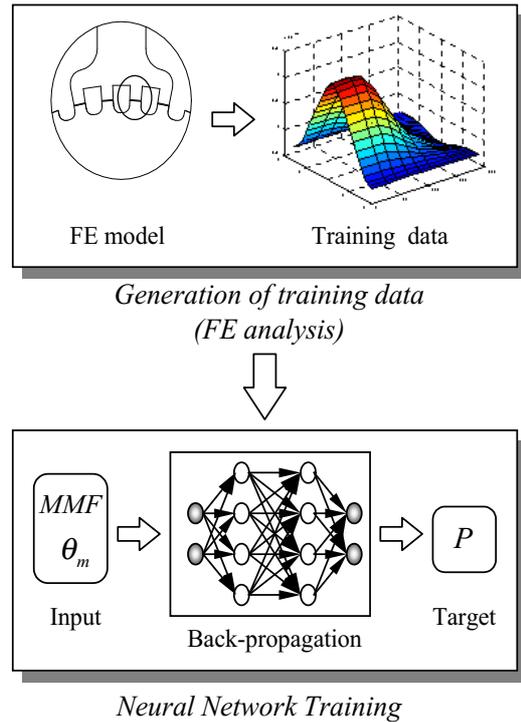


Fig. 11. Off-line neural network training process

TABLE I

SPECIFICATIONS OF TEST MOTOR

Item	Value	Unit
Number of phases	5	
Rated current	0.75	(A)
Rated holding torque	0.24	(Nm)
Turns per pole	53	
No. of teeth per stator pole	4	
No. of rotor teeth	50	
Airgap length	0.05	(mm)

Fig. 14 shows the static torque characteristics of the test motor at the rated current which are calculated for each incremental rotor position.

In Fig. 14, the conventional method describes the case that the permeance between the stator pole and the rotor is considered as the airgap permeance alone. The advanced method is to take the required permeance from the tooth/airgap model illustrated in Fig. 6 and has the sufficient accuracy for motors where saturation is confined to the tooth/airgap region.

The conventional method estimates the maximum value of the static torque distribution, which is defined as the holding torque, to be approximately 2 times greater compared with the experimental one and the advanced method shows the more reasonable result corresponding to the experimental value.

Fig. 15 illustrates the holding torque to the exciting

current characteristics. As results, the conventional method shows the results that have a linearity in all current region and the advanced method shows the saturation of the analysis results with the current increasing because of the saturation in the stator/rotor tooth region by which the developed torque is limited.

Fig. 16 shows the cogging torque with respect to the rotor position computed by the advanced method.

In general, the cogging torque is susceptible to the accuracy of the circuit parameters and difficult to measure the reliable results experimentally because of relatively small value.

When the exciting current is increased over certain current level, the developed torque is on the decrease due to the saturation of the stator pole as well as the tooth. And, the saturation of the stator pole can be easily considered by adding the non-linear reluctance of the stator pole to the equivalent circuit.

Fig. 17 shows the holding torque characteristics with respect to the stator pole width. As indicated in Fig. 17, the narrow pole width directly leads to the reduction of the holding torque.

### VIII. CONCLUSION

This paper proposes the advanced analysis technique by using the equivalent magnetic circuit taking into account the saturation effect of both teeth and poles of 5-phase hybrid stepping motors, in which the FEM is used to calculate the accurate circuit parameters. In addition, the neural network is used in combination with the FEM results to estimate the circuit parameters. As results, the solution time for calculating the circuit parameters using the neural network estimator is dramatically reduced, while sufficient accuracy is still provided.

The proposed analysis technique can be easily applied to the design process and optimization problem in case that the tooth/airgap geometry is determined in advance.

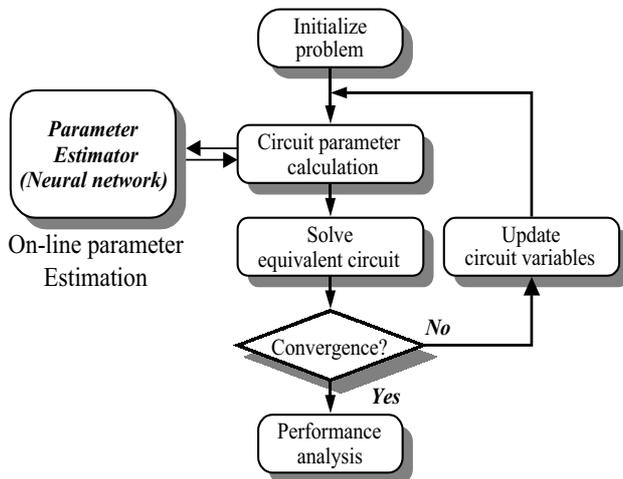


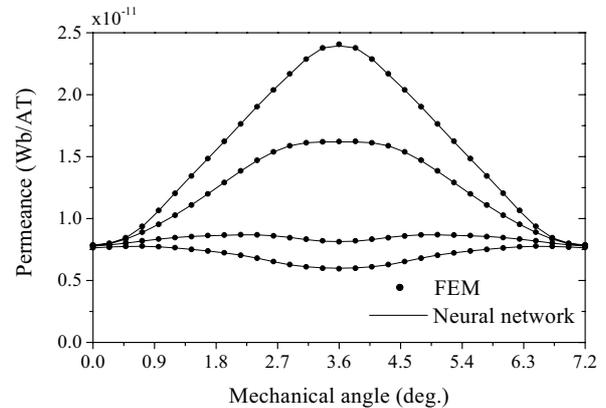
Fig. 12. Improved process of performance analysis

### ACKNOWLEDGMENT

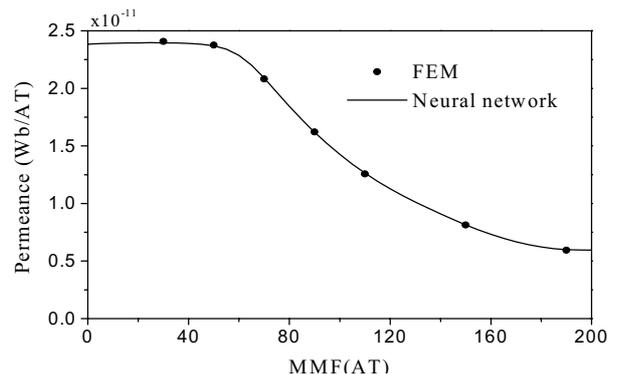
This work was supported by the Korea Science and Engineering Foundation (KOSEF) through the Machine Tool Reach Center at Changwon National University

### REFERENCES

- [1] M. K Jenkins, D Howe, T S Birch, "An improved design procedure for hybrid stepper motors", IEEE Transactions on Magnetics, Vol.26, pp.2535-2537, 1990.
- [2] Kiyonobu Mizutani, Shigeo Hayashi, Nobuyuki Matsui, "Modeling and Control of Hybrid Stepping Motors", in proc. of IEEE/IAS Annual Meeting, 1993.
- [3] Nobuyuki Matsui, Makoto Nakamura, Takashi Kosaka, "Instantaneous Torque Analysis of Hybrid Stepping Motor", IEEE Transactions on Industry Applications, Vol. 32, pp.1176-1180, 1996.
- [4] Ping Zhou, John Gilmore, Zsolt Badiacs, Zoltan J. Cendes, "Finite Element Analysis of Induction Motors Based on Computing Detailed Equivalent Circuit Parameters", IEEE Transactions on Magnetics. Vol. 34, pp.3499-3502, 1998.
- [5] Benjamin C. Kuo, "Theory and Applications of Step Motors", 1974.
- [6] Kamel Idir, Liuchen Chang, Heping Dai, "Improved Neural Network Model for Induction Motor Design", IEEE Transactions on Magnetics, Vol. 34, pp.2948-2951, 1998.



(a) Rotor position versus permeance



(b) MMF drop versus tooth airgap permeance

Fig. 13. Comparison of FEM and Neural network results

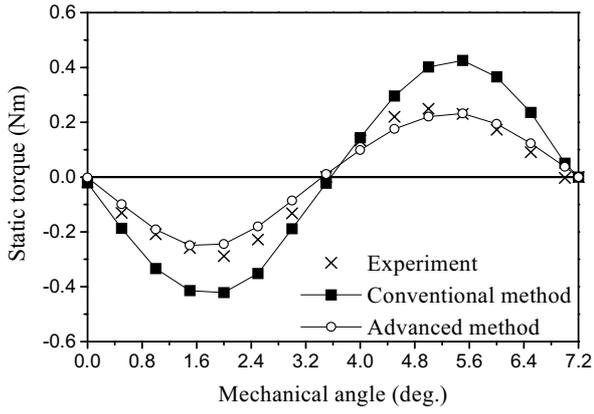


Fig. 14. Static torque characteristics

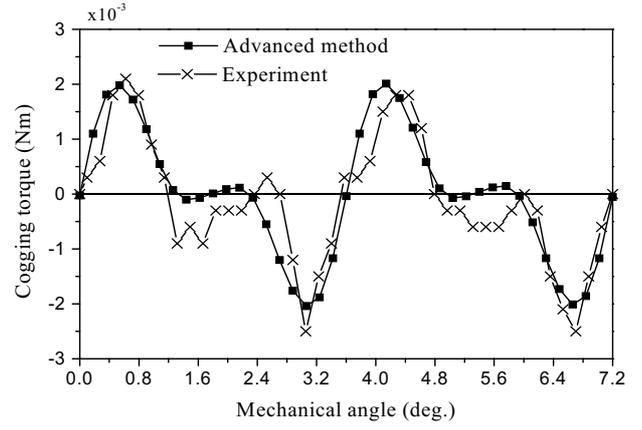


Fig. 16. Cogging torque characteristics

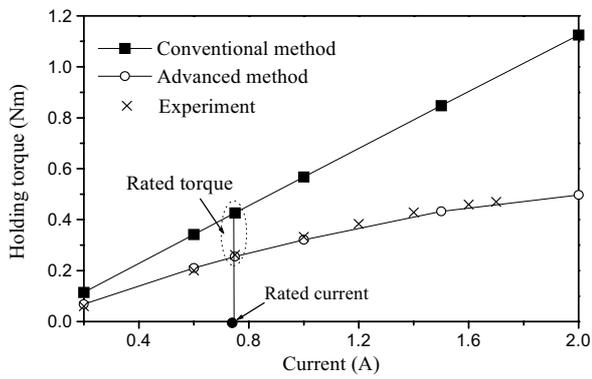


Fig. 15. Holding torque versus exciting current characteristics

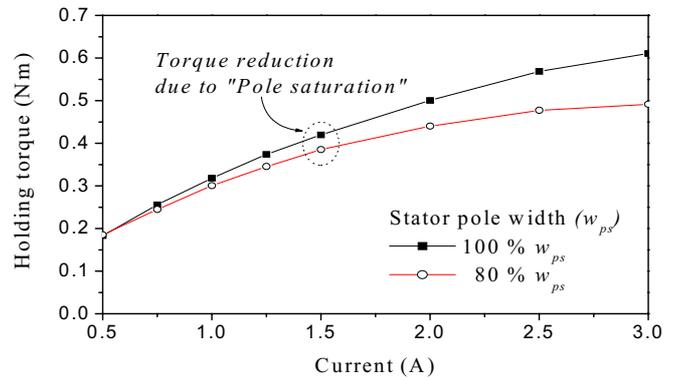


Fig. 17. Holding torque versus stator pole width