

# Application of Response Surface Methodology to Robust Design of BLDC Motor Performance

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**Abstract**—This paper describes an approach to robust design of brushless dc motors considering the cogging torque reduction and running torque optimization to enhance the robustness. The approach is based on the response surface methodology and the estimated model is used to minimize the total sensitivity of design variables. A validity of this approach is verified through comparing the robust solution with the normal result obtained by a conventional optimization procedure.

## INTRODUCTION

Generally, a design problem is a natural process to optimize a solution versus a specified requirement. The problem can be complex because there are many numbers of design variables and these design variables frequently interact with each other. Moreover, when designing a certain motor by using a conventional optimization algorithm, their performance cannot be satisfied as the desired one in certain cases. It is due to the limitations on the manufacturing tolerances and job requirements that couldn't be considered, adjustments for improving the efficiency of manufacturing, and experiences of a skilled labor etc. In general, the variation in manufacturing processes can affect the machine performance, such as operating efficiency, reliability, and production of vibration [1]. Therefore, it is desired to calculate the robust solution.

Recently, the demand for BLDC motor is expanding rapidly. Some of applications in the industry are more demanding and requiring a good quality. BLDC motor is to be designed to perform in accordance with design specifications. The cogging torque of BLDC motor arises from the interaction between its rotor magnet and slotted stator. It exerts a bad influence upon on the motor performance. Therefore, this paper illustrates robust design to reduce the cogging torque. An optimization technique based on the response surface methodology is applied to robust design of BLDC motor performance. The robust design is achieved by minimization of the total sensitivity concerning design variables.

The response surface methodology is well adapted to make an analytical model for a complex problem [2]-[3]. Moreover, the response surface provides the designer with an overall perspective of the system response according to the behavior of design variables within a design space. It can be leading to great savings in time and efficiency without very large repetition and expensive of computations. In this paper, a computation of the total sensitivity concerning design variables is achieved by using analytical model obtained from

the response surface methodology.

## DESIGN OF EXPERIMENT AND FORMULATION

### Identify Design Variables

The stator has 18 slots and the rotor is built of six tiles of radial permanent magnet. Major parameters of its geometry, such as lamination stack height is 30 mm and magnet thickness is 3 mm, are obtained by the procedure of a initial design. During the optimization procedure to reduce the cogging torque, the design focused on the influence of the rotor magnet and slotted stator, therefore three design variables define the shape of BLDC motor as shown in Fig. 1.

### Concept of The Response Surface Methodology

The response surface methodology procedures seek to find the relationship between design variable and response through statistical fitting method [3], which is based on the observed data from the process or system. The response is generally obtained from real experiments or computer simulations. Thus, computer simulations are performed in this paper. We suppose that the true response can be written as

$$\eta = F(\zeta_1, \zeta_2, \dots, \zeta_k) \quad (1)$$

Where, the variables  $\zeta_1, \zeta_2, \dots, \zeta_k$  in (1) are expressed in natural units of a measurement, so called the natural variables. Because the form of the true response function  $F$  is unknown and perhaps very complicated, we must approximate it. In many cases, the approximating function  $y$  of the true response  $F$  is normally chosen to be either a first-order or a second-order polynomial model, which is based on Taylor series expansion.

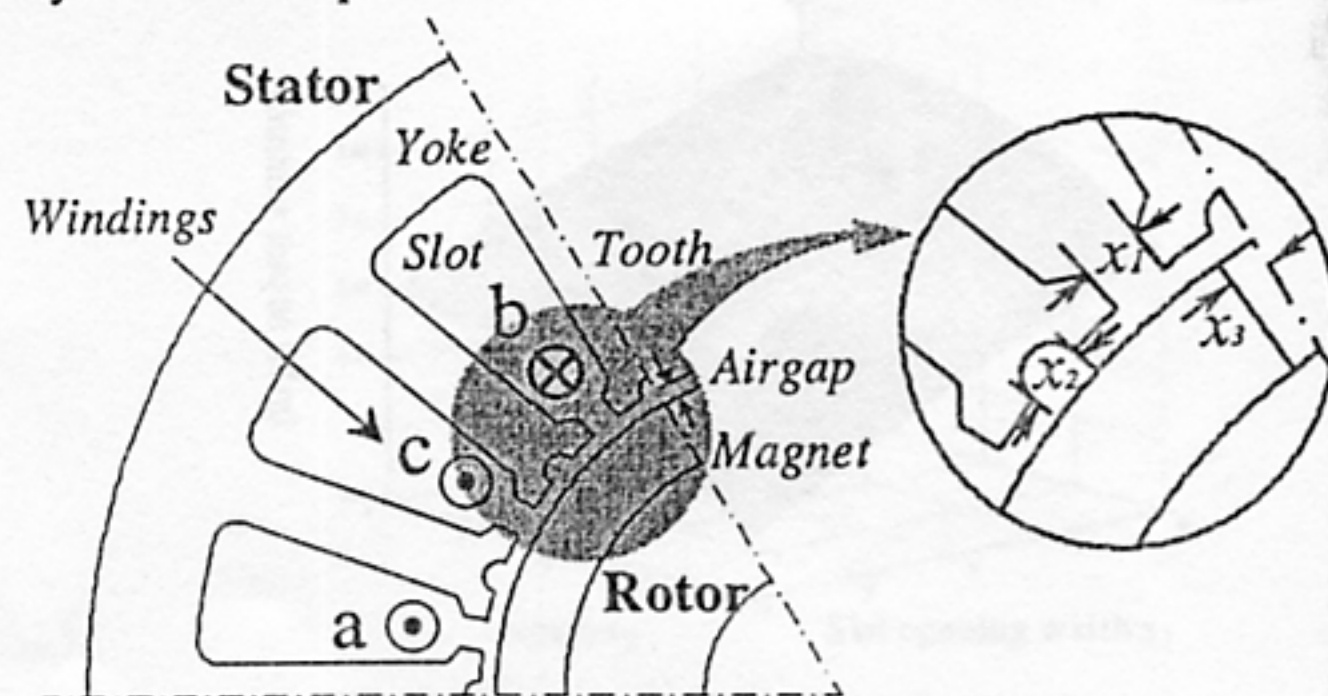


Fig. 1. Analysis model and design variables



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In general, the first-order model is

$$y = \beta_0 + \beta_1 x_1 + \dots + \beta_k x_k \quad (2)$$

and the second-order model is

$$y = \beta_0 + \sum_{j=1}^k \beta_j x_j + \sum_{j=1}^k \beta_{jj} x_j^2 + \sum_{i,j=1}^k \beta_{ij} x_i x_j \quad (3)$$

where,  $\beta$  is regression coefficients. In order to more accurately predict the response, the second-order model is used to fit a curvature response at this paper. The regression coefficients of the predictive model are estimated by the method of the least squares using the general formulation as

$$\mathbf{b} = (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T \mathbf{Y} \quad (4)$$

Where,  $\mathbf{b}$  is the matrix of estimated coefficients,  $\mathbf{X}$  is the matrix of model terms,  $\mathbf{X}^T$  is the transpose of the matrix  $\mathbf{X}$ , and  $\mathbf{Y}$  is the matrix of the observed response. Using the estimated coefficient, the second-order predictive model or analytical model can be build like as (2)

#### Optimization Problem and Robust Design

The general formation of a conventional optimization is expressed as following [4].

$$\text{Minimize: } f(x_i), \quad i = 1, 2, \dots, k \quad (5)$$

$$\text{Subject to: } g_j(x_i) \leq 0, \quad j = 1, 2, \dots, m \quad (6)$$

$$x_{il} \leq x_i \leq x_{iu}, \quad k = 1, 2, \dots, n \quad (7)$$

where,  $f(x)$  is the objective function,  $g_j(x)$  is the constrain functions with the dimension of  $m$  and  $x_i$  is  $k$  design variables with lower and upper bounds of  $x_{il}$  and  $x_{iu}$ , respectively. The goal of the optimization is set as to achieve both reduction of the cogging torque and satisfaction of the running torque. The analytical model built from the response surface methodology can be use in a optimization procedure as objective or as constraint. The analytical model of the cogging torque is used in a objection function and The analytical model of the running torque is used for a constraint function, respectively.

The next step in the design is to minimize the total sensitivity of design variables. In order to obtain the robust optimal solution, the original objective function can be replaced to following described function.

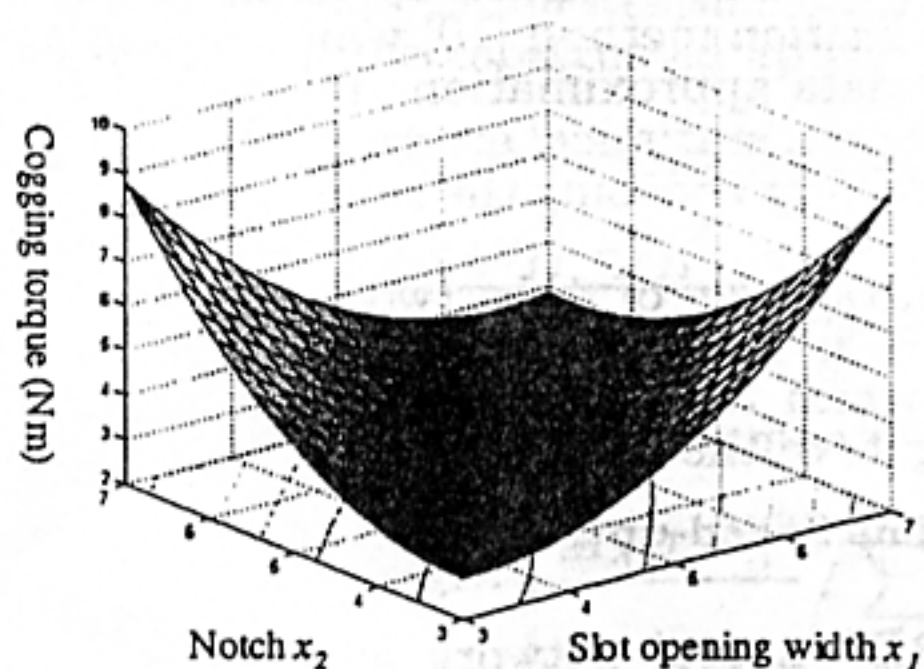


Fig. 2. Response surface of cogging torque.

$$\text{Minimize: } \varphi(x) = \sum_{i=1}^k \left| \partial(f(x) - f_0)^2 / \partial x_i \right| \quad (8)$$

where  $f_0$  is the value of the cogging torque obtained by the previous optimization procedure.

The sequential quadratic programming method has been commonly used to minimize the objective function that satisfies the constraint at this paper.

#### RESULTS AND DISCUSSION

The response surfaces of the cogging and running torque are shown in Fig. 2 and Fig. 3. In order to make precise analytical model, the peak-to-peak value of the cogging torque is multiplied by 1000 and taken natural logarithm. The running torque is a rms value. It is confirmed that the robust solution less insensitivity than normal optimum point.

#### CONCLUSIONS

This paper presents robust design with minimizing the total sensitivity of design variables combined with the response surface methodology. Therefore, it is expected that the proposed robust design can be easily utilized to enhance the product robustness.

#### ACKNOWLEDGMENT

This work has been supported by G7 (Next generation vehicle project), which is funded by Ministry of commerce, Industry and Energy and Ministry of Science & Technology

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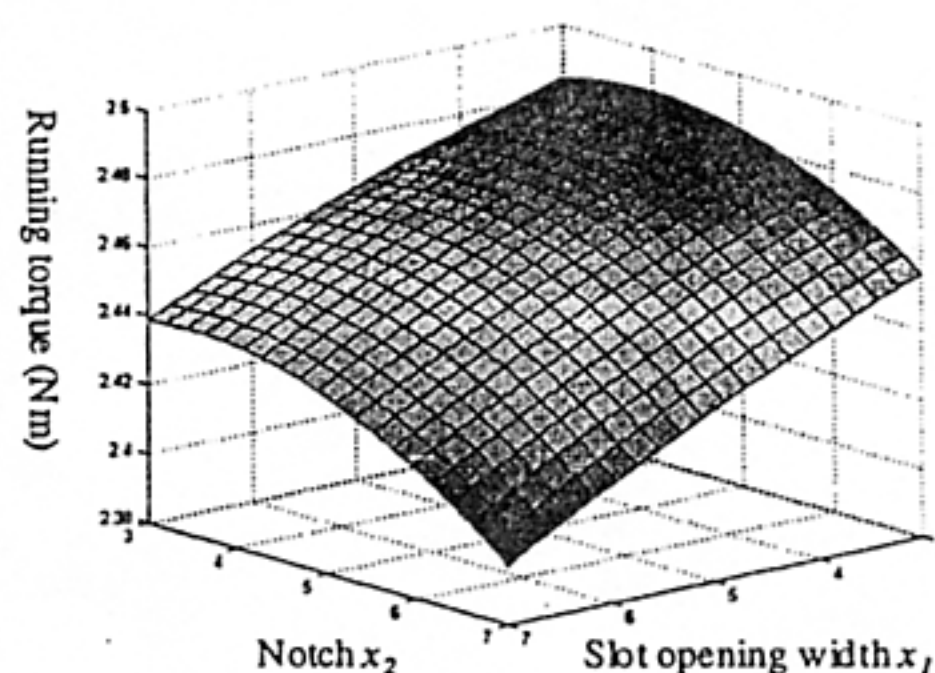


Fig. 3. Response surface of running torque.