

# Dynamic analysis of eddy current braking system using FEM

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**Abstract** — The dynamic analysis of eddy current braking system is studied. The transient analysis of eddy current brake with a finite conductor is solved using 2-D FEM. The moving distance, speed of conductor and dragging force from eddy current are calculated as functions of time. The mechanical equation and electromagnetic equation are coupled assuming that the eddy current dragging force plays a role as the nonlinear damper. In order to express the motion of conductor in FEM, the moving mesh technique is used. The result of analysis is compared with the experiment.

## INTRODUCTION

In this paper, the dynamic analysis of a linear eddy current braking system is studied. The eddy current develops under the situation that some sources (current, voltage, flux and so on) vary as the time changes and that the relative motion of conductor and flux happens. The dynamic characteristics of conductor are included in the eddy current problem such that the motion of conductor cause every variables (speed, dragging force, moving distance of mover) change in the eddy current braking system. Until now, many researches focused on the eddy current braking system with infinite conductor (rail) at constant speed[1, 2, 3]. And, using FEM, the various modified methods were tried to solve the eddy current problem with the relative motion of conductor and flux.[3, 4] In this paper, to understand the dynamics of eddy current brake operation, the prediction of the stopping time and the displacement of conductor at every time-step is carried out. In this study, 2-D FEM and the moving mesh technique to express the motion of mover are used. The result of analysis is compared with experimental one.

## PROBLEM FORMULATION

### The basic equations

The following equations explain the principal Maxwell equations considering the eddy current.

$$\nabla \times \mathbf{H} = \mathbf{J}_s + \mathbf{J}_e \quad (1)$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} + \nabla \times (\mathbf{v} \times \mathbf{B}) \quad (2)$$

$$\nabla \cdot \mathbf{B} = 0 \quad (3)$$

$$\mathbf{B} = \mu \mathbf{H} \quad (4)$$

where  $\mathbf{J}_s$ ,  $\mathbf{J}_e$ ,  $\mathbf{B}$ ,  $\mathbf{v}$ ,  $\mu$  are the source current, the eddy current which generated in the moving conductor, the flux density, the speed of moving conductor, and the permeability of magnetic materials, respectively.

Fig.1 shows the simple configuration of three regions expressing the eddy current problems with motion.

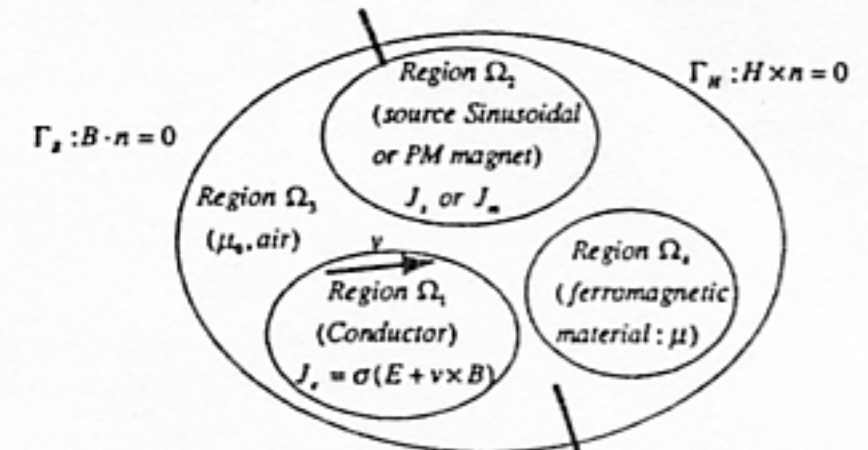


Fig. 1. Eddy current problem with the moving conductor

The three boundary conditions of eddy current problem is explained by (5).

$$\begin{aligned} \mathbf{B}_1 \cdot \mathbf{n} &= \mathbf{B}_3 \cdot \mathbf{n} \\ \mathbf{H}_1 \times \mathbf{n} &= \mathbf{H}_3 \times \mathbf{n} \\ \mathbf{J}_e \cdot \mathbf{n} &= 0 \end{aligned} \quad (5)$$

Even though the current vector potential is used to analyze the eddy current problem, in this paper, the magnetic vector potential is used as variable.

$$\mathbf{B} = \nabla \times \mathbf{A} \quad (6)$$

Using (1)~(4) and (6), the governing equation for overall system can be derived as follows

$$\nabla \times \left( \frac{1}{\mu} \nabla \times \mathbf{A} \right) = \mathbf{J}_s + \sigma \left( -\frac{\partial \mathbf{A}}{\partial t} + \mathbf{v} \times \nabla \times \mathbf{A} \right) \quad (7)$$

### Formulation for transient analysis

The governing equation can be formulated as the matrix form using Galerkin Method, Green Theorem and the derivative formula.

$$[\mathbf{S}]^T [\mathbf{A}]^e + [\mathbf{T}]^T \frac{d}{dt} [\mathbf{A}]^e = [\mathbf{F}_s] \quad (8)$$

In analyzing the time derivative term in (8), the time difference method is used. Using the backward difference approximation, the final discretized matrix equation is

$$[\mathbf{S}]_{t+\Delta t}^e [\mathbf{A}]_{t+\Delta t}^e + \frac{[\mathbf{T}]^e ([\mathbf{A}]_{t+\Delta t}^e - [\mathbf{A}]_t^e)}{\Delta t} = [\mathbf{F}_s] \quad (9)$$

In order to express the motion of mover or conductor, the moving mesh technique is used in FEM [5].



## COUPLING SYSTEM

## CONCLUSION

*The dynamic equation on moving conductor*

The mechanical term has been considered to understand the mechanical motion besides the motion of electrical device [6]. Most of studies have focused on the motion of rotor in the electromechanical system using the dynamic equation [1, 5].

The motion of the conductor can be calculated by solving the dynamic equation and the Maxwell Stress Tensor at each time step. Assuming that the load force is negligible.

$$m\ddot{x} + c\dot{x} + \mathbf{f} = F \quad (10)$$

$$c\dot{x} = \mathbf{f}_{\text{eddy}} \quad (11)$$

where  $F$ : external force,  $m$ : mass of conductor,  $c$ : nonlinear damping coefficient by eddy current,  $\mathbf{f}$ : friction force,  $\mathbf{f}_{\text{eddy}}$ : dragging force by eddy current,  $\dot{x}$ : speed of conductor [m/s]

Using the time backward difference method, the speed is

$$v^{t+\Delta t} = v^t + \frac{-(\mathbf{f}_{\text{eddy}}^{t+\Delta t} + \mathbf{f})\Delta t}{m} \quad (12)$$

## NUMERICAL EXAMPLE

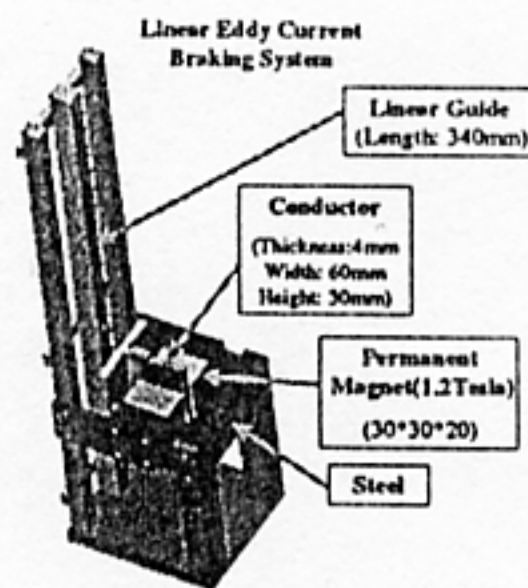


Fig. 2. System structure

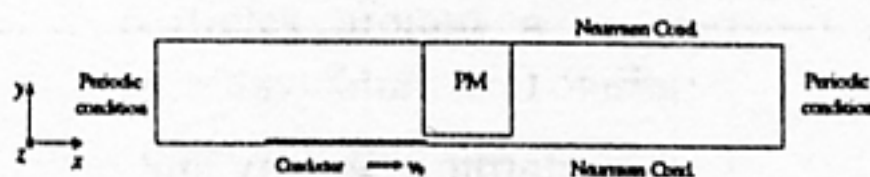


Fig. 3. 2-D FE model

The system for analysis is modeled considering symmetric condition in 2-D FEM as shown in Fig. 2 and Fig. 3. When entering speed is 1.22[m/s], the final moving distance in experiment and analysis is 40[mm] and 40.32[mm], respectively. From Fig. 4 to Fig. 7, considering the friction force of linear guide, the force and acceleration of mover increase rapidly and decrease smoothly as time goes. As the conductor moves, the speed of it decreases and increment of the moving distance decreases.

In this paper, the dynamic analysis of the eddy current braking system with a moving finite conductor is studied using 2-D FEM and moving mesh technique. The coupling equation is applied to express the dynamics of system. The results are very similar to experimental data. This dynamic analysis can predict well the system's dynamic variables considering the eddy current until the system stops.

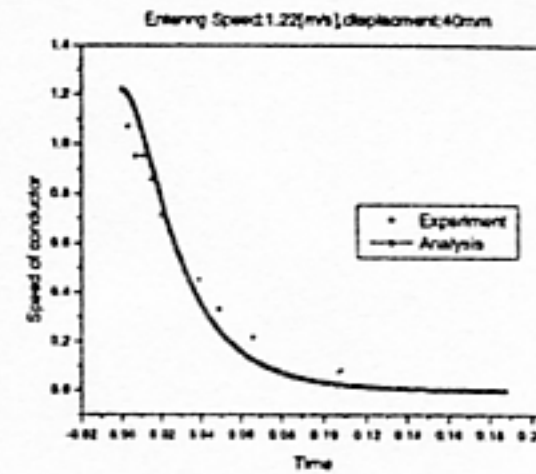


Fig. 4. Comparison of speed

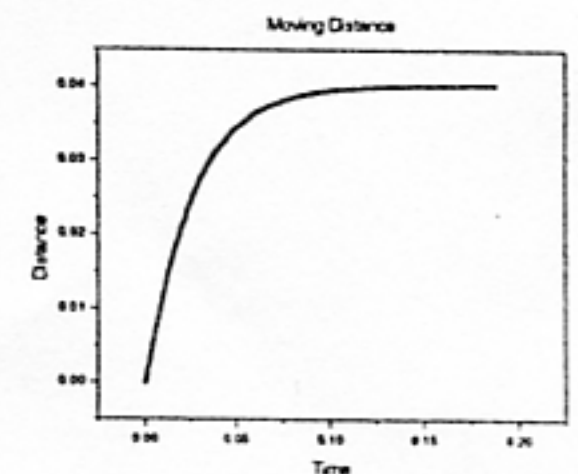


Fig. 5. Moving Distance of mover

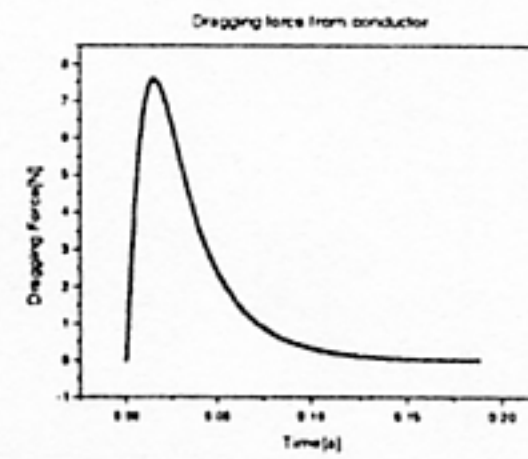


Fig. 6. Dragging force

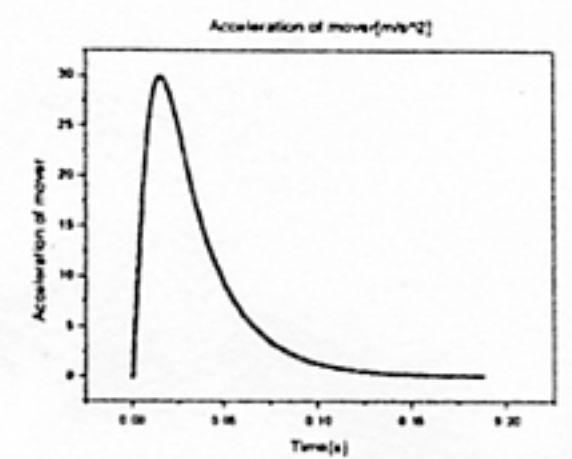


Fig. 7. Acceleration of mover

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