ANALYSIS OF ROTOR DYNAMIC BEHAVIOUR DUE TO UNBALANCED MAGNETIC FORCE IN SRM

Kyung-Ho Ha Jung-Pyo Hong
Dept. of Electrical Eng., Changwon National University, Changwon, Kyungnam, 641-773, Korea
Phone: +82-551-279-7519, Fax: +82-551-263-9956, E-mail: haroom@netian.com and jphong@sarim.changwon.ac.kr

Abstract: *In order to reduce vibration of SRM, it is necessary to predict the mechanical vibration behaviour of rotor-bearing system. The purpose of this paper is to analyze the rotor behaviour due to the unbalance electromagnetic force and unbalance mass in Switched Reluctance Motor (SRM). The static and dynamic eccentricity with the rotation of rotor is analyzed by using the mechanical problem coupled with the electromagnetic filed. The electromagnetic force acting on the rotor is calculated by Maxwell stress tensor. In addition, the frequency response function as a function of the speed of revolution is investigated to know the resonance speeds.*

Key words: *Vibration behavior, SRM, static and dynamic eccentricity and resonance speed*

1. Introduction

SRM has wide application fields since it exhibits several positive advantages over conventional AC or DC machines. It is simple and robust structure, high performance for speed control and high efficiency for wide speed range. However, as the most disadvantages of SRM are a high level of the vibration and torque ripple, there is the difficulty of a problem in the use of the commercial products.

Among many possible sources of vibration and noise in SRM, the relationship between magnetic and mechanical origins is to be focused. The commutation of the tangential forces which are exerted on the poles of the stator and the rotor and produce torque, can excite the vibration of the stator and rotor. And normal SRM operation is characterized by radial forces on opposite poles, which tend to deform the stator into oval. A vibration stator mode will be excited when the radial forces on the stator poles coincide with the natural vibration.

In a rotor vibration point of view, manufacturing asymmetries of the rotor such as a non-uniform air-gap and unbalance mass, excites asymmetric forces on the rotor. This unbalance mechanical and electromagnetic force lead to the eccentricity of rotor and deteriorate the performance of SRM. Therefore, it is important to estimate the mechanical behavior caused by electromagnetic force to reduce the vibration and acoustic noise [1,2]. However, the existing research deals with the stator vibration and control strategy to reduce the vibration and noise.

The rotor eccentricity takes two forms, which is static and dynamic one [1]. The rotor is displaced within always the constant eccentricity when the rotor is rotating hence the unbalance magnetic force is a steady force. For instant, the residual unbalanced mass leads to centrifugal force that is related to the static eccentricity. However, as the unbalance magnetic force in SRM is not uniform but changing with rotor position, it causes the dynamic eccentricity. Accordingly, it is necessary to predict the rotor eccentricity in order to prevent serious all the possible operational problems [2].

2. Theoretical approach

This paper deals with the analysis of the rotor vibration based on the structure analysis combined with electromagnetic force. The finite element method is used as the method of both analysis fields. The rotor vibration in this paper set limit to the static and dynamic eccentricity caused by the unbalance mass of rotor and the electromagnetic force, as well as the rotor response as a function of revolution.

In an analysis of the dynamic eccentricity, as it is obtained by a transient structure analysis, time stepping approach used in the differential motion equation is required. Several numerical methods are available for the solution of vibration problem. The Houbolt algorithms coupled with FEM is used to obtain the dynamic eccentricity as a function of time due to unbalanced force, which is solved by a step by step procedure with respect to time [3]. The unbalance magnetic force acting on the rotor is analyzed by Maxwell stress tensor. Based on this method, the dynamic eccentricity and the static eccentricity caused by the electromagnetic force is compared with the static eccentricity due to unbalanced mass. In addition, the frequency response function to avoid the resonance is investigated by the simulation, which is derived by frequency response function.

3. Analysis model and Procedure

The analysis model is a SRM with 4 rotor poles and 6 stator poles with three phases winding in the stator. The air-gap is 0.6(mm). Table 1 presents the measured mechanical properties needed for analysis of mechanical structure.

Table 1
Mechanical material property

Core specific density	7310 (kg/m^3)	Shaft specific density	7850 (kg/m^3)
Bearing stiffness	5.36×10^5 (N/m)	Young's Modulus	205 (Gpa)

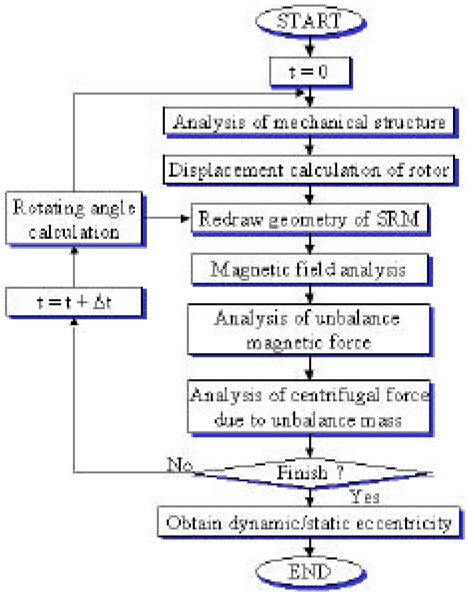


Fig. 1 Analysis of rotor dynamic eccentricity

Fig. 1 shows the analysis procedure of time step to solve the differential motion equation for forced vibration in transient state and it is explained as follows:

(i) **Step 1**: The displacement of rotor due to unbalanced force at the instant t is calculated by structure analysis.

(ii) **Step 2**: The analysis model for magnetic field analysis, based on the displacement results of previous step at the rotor position matching with time step, is redrawn.

(iii) **Step 3**: The unbalanced magnetic force in rotor is obtained by solution of magnetic fie ld problem at the instant time t.

(v) **Step 4**: Calculation centrifugal force due to unbalance mass.

(v) **Step 5**: The processes from Step 1 are repeated.

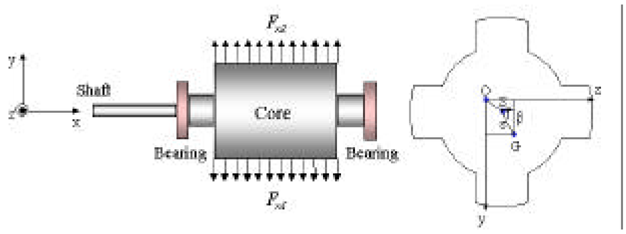
4. Force calculation

A. Unbalance electromagnetic force

The unbalanced electromagnetic force F_e due to eccentricity is obtained by the difference of F_{n1} and F_{n2} as shown in Fig. 2(a). The governing equation of the magneto-static field problem with magnetic vector potential $\boldsymbol{A}$ is generally expressed as follows:

$$\nu\nabla^2\boldsymbol{A} = \boldsymbol{J} \quad (1)$$

where n is magnetic reluctivity, J is the applied current.



(a) Unbalance electromagnetic force
(b) Centrifugal force due to unbalance mass
Fig. 2. Unbalance force due to electromagnetic force and mass

The Maxwell stress tensor from the magnetic field analysis is used in order to obtain the magnetic force acting on the rotor. The surface force density p on a node i for ferromagnetic material is represented as follows:

$$p = (1/m_0)\cdot[(\vec{n}\cdot\vec{B})\cdot\vec{B} - (1/2)\cdot(\vec{B}\cdot\vec{B})\cdot\vec{n}] \quad (2)$$

where $\vec{n}$ is direction of the normal unit vector on the pole face. $\vec{B}$ is flux density.

B. Unbalance mass force

The rotors have always some amount of residual unbalance however well they are balanced, which develops the centrifugal force with rotation speed that makes the static eccentricity. The geometric center of rotor is S and its center of gravity is at a distance e called eccentricity. In the fixed OXY axis system, the geometry of unbalance whirl at the rotor is shown in Fig. 2(b).

The mass unbalance U is defined as product of the unbalance mass m and eccentricity e, the centrifugal force F_m in x-y plan when the eccentric rotor rotating wt is then expressed as equation (4) [4].

$$U_x = m\varepsilon\cos\beta, U_y = m\varepsilon\sin\beta \quad (3)$$

$$F_m = m^2 U_y \cos\omega t + m^2 U_x \sin\omega t \quad (4)$$

Accordingly, the total unbalance force acting on the rotor surface is the sum of centrifugal force and magnetic force.

5. Structural analysis of rotor vibration

A. Static vibration

Based on this finite element model, the matrix equations of motion for a beam element can be written as [5]

$$([M_t]+[M_r]^e)\{\ddot{q}(t)\}^e - w[G]^e\{\dot{q}(t)\}^e + [K]^e\{q(t)\}^e = \{F(t)\}^e \quad (5)$$

where $[M]^e$, $[G]^e$ and $[K]^e$ are the element mass, gyroscopic and stiffness matrices (8 by 8), respectively. $q(t)$ is the vector of node displacement. The exciting force term $F(t)$ includes the unbalanced magnetic force or centrifugal force. The components of each matrix are shown in reference [5].

The beam element used in this paper has four Degrees of Freedom at each node. The state variables of one beam element is expressed as :

$$q = [x_1 \quad q_1 \quad y_1 \quad f_1 \quad x_2 \quad q_2 \quad y_2 \quad f_2] \qquad (6)$$

where, x: x direction deflection, q: x direction angle deflection, y: y direction deflection, f: y direction angle deflection. The superscripts 1, 2 in equation (6) is the number of node point at both end of the beam element. As the static eccentricity is independent of time, the amount of static eccentricity is obtained by solving equation (5).

B. Dynamic vibration

For the analysis of the equation (5), a numerical approach, the Houbolt algorithm gives the required high stability and rapid convergence in the case of a large time step. The second-order differential equation (5) combined with this algorithm result in an equation, which needs to be solved to find the displacement of rotor.

The step by step procedure is given below [6].

From the known initial conditions $q(t=0)=q_0$ and $\dot{q}(t=0)=\dot{q}_0$, find $\ddot{q}(t=0)=\ddot{q}_0$ using equation (6).

$$\ddot{q}_0 = [M]^{-1}\cdot(F_0 - [G]\dot{q}_0 - [K]q_0) \qquad (7)$$

Select a suitable time step Dt

Determine q_{-1} using equation (8).

$$q_{-1} = q_0 - \Delta t\cdot\dot{q}_0 + (\Delta t)^2/2\cdot\ddot{q}_0 \qquad (8)$$

Find q_1 and q_2 using the central difference equation (9)

$$q_{i+1} = [(1/(\Delta t)^2)\cdot[M] + (1/2\Delta t)\cdot[G]]^{-1}\cdot [F_i - ([K] - (2/(\Delta t)^2)\,[M])\cdot\dot{q}_i - ((1/(\Delta t)^2)\cdot[M] - (1/2\Delta t)\cdot[G])\cdot q_{i-1}]. \qquad (9)$$

Compute q_{i+1}, starting with the i=2 and using equation (10).

$$\begin{aligned} q_{i+1} = {} & [(2/(\Delta t)^2)[M] + (11/6\Delta t)[G] + [K]]^{-1} \\ & \times[F_{i+1} + ((5/(\Delta t)^2)[M] + (3/\Delta t)[G])q_i \\ & - ((4/(\Delta t)^2)[M] + (3/2\Delta t)[G])q_{i-1}\times \\ & ((1/(\Delta t)^2)[M] + (1/3\Delta t)[G])q_{i-2}]. \end{aligned} \qquad (10)$$

The nodal displacement at subsequent time instants, which is the eccentricity of rotor according to a change of time, has been solved by repetitively above mentioned the same steps.

6. Analysis results and discussion

The mesh of analysis model, shown in Fig. 3, is composed of the 23 beam elements used for structural FEM. The stiffness for two elements placed on the bearing part of the rotor is applied to the third term in equation (5). The basic assumption to simplify analysis is that the bearing and stator core is isotropic and homogeneous.

In the analysis of magnetic field, the problem takes into account nonlinear due to steel's magnetic characteristics. The core of rotor is loaded by the unbalance electromagnetic force that is beginning from the centrifugal force due to unbalance mass of asymmetrical rotor, when the rotor is rotating.

Fig. 4 shows the static eccentricity caused by only unbalanced mass 10(g·mm). This results is calculated at point A shown in Fig. 3 when the rotor speed is 2000(rpm). Fig. 5 represents the 3-dimensional whirling motion of rotor under the static eccentricity.

Fig. 6 shows the dynamic eccentricity in transient state due to electromagnetic force with unbalanced mass under the same condition. The displacement of rotor with the change of time, which means a locus of the geometry center of rotor, is displayed on x-y plan from 0 to 1(sec) per time step 0.0002(sec).

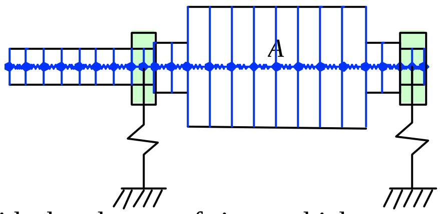


Fig. 3. Mesh of analysis model

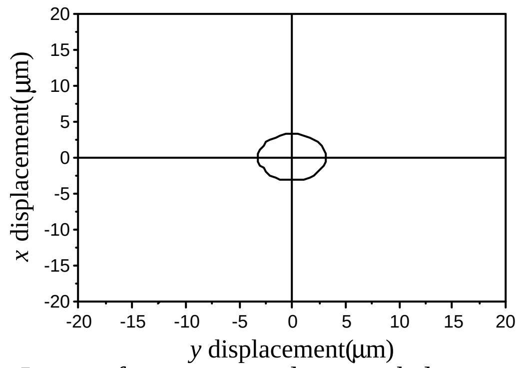


Fig. 4. Locus of rotor center due to unbalance mass in steady state

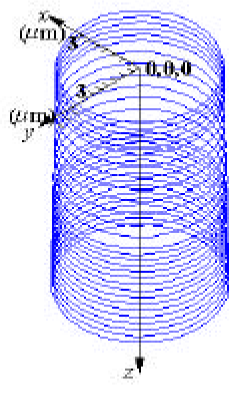


Fig. 5. 3D whirling motion of static eccentricity

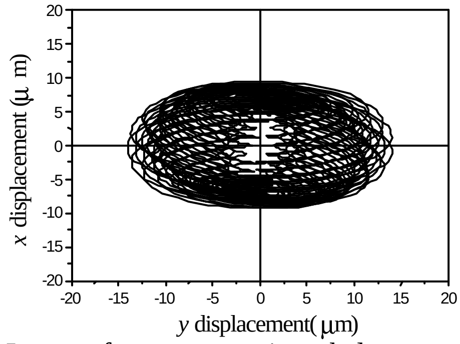


Fig. 6. Locus of rotor center in unbalance mass and electromagnetic force in transient state

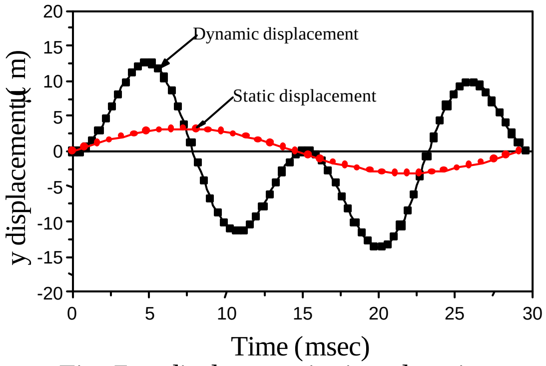


Fig. 7. y-displacment in time domain

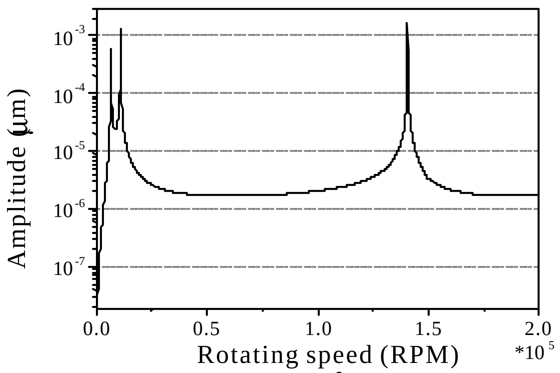


Fig. 8. Frequency response function at point A

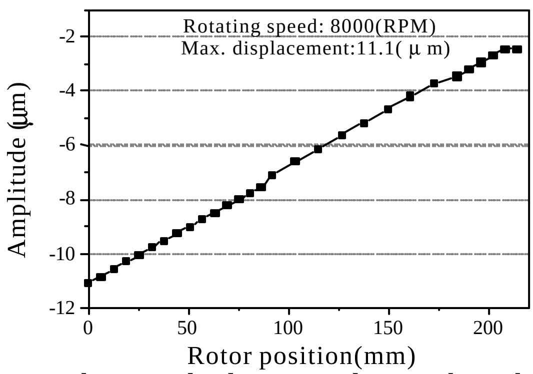


Fig. 9. The rotor displacement along with stack

Fig. 7 shows the y displacement of rotor during one rotation in time domain. The amplitude of y displacement with magnetic force is estimated about 4 times larger than that without magnetic force. It is noticed that the locus of rotor center with the magnetic force has an unstable tendency to turn around the stator bore center without regular period. Therefore, it is verified that the unbalanced magnetic force affects the eccentricity of rotor and increases the displacement as well

Fig. 8 shows the displacement at point A caused by unbalance mass 10(g·mm) according to the rotation speed. This result note that the resonance speed is 109.34, 2250.09 and 187.37(Hz) based on the peak point of Fig. 8.

Fig. 9 shows the displacement along with rotor stack length when the rotor speed is 800(rpm). The maximum displacement of rotor is 11.1(μm) in front of shaft.

7. Conclusions

It is necessary to predict the rotor eccentricity to prevent vibration. The static and dynamic eccentricity caused by the unbalance force is analyzed by the structural FEM coupled with the electromagnetic FEM. The resonance speed possible is obtained from the frequency response function. This analysis method proposed in this study is useful in designing mechanical rotor, balancing rotor and diagnosing fault on operation.

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