

Resonance Frequency and Vibratory Behaviour of Rotor in Switched Reluctance Motor

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Abstract-- This paper deals with the natural vibration modes and frequencies of the rotor in a 6/4 Switched Reluctance Motor (SRM). The effect of the mode shapes of the rotor on the vibration characteristics is investigated. Transfer Matrix Method (TMM) is used to calculate the 3-dimensional whirling modes and the natural frequencies. It is necessary to know the resonance speed and the mode shapes to reduce the vibration and noise in design stage. TMM is useful to predict the vibration behavior of rotor because of a rapid computing time. The analysis results based on TMM are validated by the comparison with that obtained by structural finite element analysis and experimental values.

Index Terms-- Behaviour of rotor, resonance speed, SRM and Transfer matrix method.

I. INTRODUCTION

SRM is a fascinating machine with all kinds of interesting application possibility because it has many advantages such as the possibility of high-speed operation and simplicity mechanical construction. However, the most striking disadvantages of the SRM are high levels of vibration and torque ripple. Among many possible sources of vibration and noise in SRM, the relationship between magnetic and mechanical origins is to be focused. The dominant sources are deformation of stator and eccentricity caused by rotor dynamics when the natural frequencies of the stator or rotor are close to the frequency of the electromagnetic force [1]. The mechanical origin for the magnetic force is classified into two parts, which are the stator and the rotor.

The commutation of the tangential forces, which are exerted on the poles of the stator and the rotor and produce torque,

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can excite vibration of the stator and rotor. In the aspects of the stator, normal SRM operation is characterized by radial forces on opposite poles, which tend to deform the stator into oval. The stator vibration mode will be excited when the radial forces on the stator poles coincide with the natural vibration [1], [2]. The modal deflection of the stator back iron as a ring can be predicted from calculating the modal analysis, free vibration.

Especially, the rotor will get into resonance when it rotates at speeds equal to the bending natural frequency and these speeds are called as critical speeds or resonance speeds [3]. Moreover, the determination of the position of bearing on the shaft, bearing stiffness and balancing technology is principally associated with rotor vibration mode. It is therefore necessary to know the natural frequencies and vibration modes of the rotor as well. These should be investigated in the design stage. The existing research on vibration mode and acoustic noise of the SRM mostly deals with the stator part of motor [1]-[6]. This paper deals with the rotor behavior to avoid resonance speed in high-speed operation.

As a general practice in the design of rotor is to determine the critical speeds and modes, a suitable analysis method is required to reduce calculation time. The Finite Element Method (FEM) is well established for structural analysis of the motor. Its inherent drawbacks, however, are its complexity and the length of time. It takes a lot of time to obtain a solution because the matrix size depends on the number of elements and the Degree of Freedom (DOF) at each node. On the other side, the method of TMM is independent to the number of element, so it can have the fixed matrix size regardless of the number of element and reduce the computing time. In order to apply TMM to this study, the rotor is modeled as several lumped masses connected by massless elastic shaft.

Based on the structural TMM and elasticity theory, 3-dimensional vibration modes and resonant frequencies of rotor-bearing system for SRM are analyzed and demonstrated in this paper. The analysis results are validated by measurements and then the computational gain of the proposed method is compared with that of 3-D FEM.

II. ANALYSIS MODEL

The analysis model SRM has 4 rotor poles and 6 stator poles with three phases winding in the stator. Table 1 presents the main specifications and the measured mechanical properties needed for analysis of mechanical structure. Fig.1

shows the configuration of the rotor and the analysis model. The analysis model of the rotor supported on bearings is divided into several lumped masses and then they are constructed by connecting the n lumped mass as massless elastic shaft.

TABLE I

SPECIFICATION OF ANALYSIS MODEL AND MATERIAL PROPERTY			
Item	Value	Item	Value
Rotor core mass density	7850 (kg/m ³)	Shaft mass density	7900 (kg/m ³)
Bearing stiffness	5.36×10 ⁵ (N/m)	Young' s Modulus	205(Gpa)

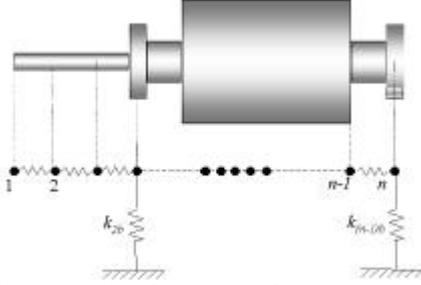


Fig. 1. Configuration and analysis model of lumped mass

III. ANALYSIS METHOD

A. Transfer matrix method

The form of the point matrix $[P]_i$, presenting the mathematical expression of the lumped masses, is derived from equilibrium relation for the mass at the i th point. The field matrix $[F]_i$, representing the massless elastic shaft, is derived from the solution of the governing differential equation of a uniform beam. The point and field matrixes for the 3-D analysis model each have eight DOF. The 8 by 8 overall transfer matrix from 1 to n th station in the over all system is developed from the product of all field and point matrix in series. The modal analysis is obtained by solving the overall transfer matrix.

The state vector that presents DOF in each element is composed of deflection w_x and w_z , angle deflection q_y and q_z , bending moment M_z and M_y , shear forces V_y and V_z in the xy and x-z planes.

- Point matrix: From the equilibrium relation for the mass at point i , we can directly write the following point transfer matrix [3].

$$\{\mathbf{S}\}_i^R = [P]_i \{\mathbf{S}\}_i^L \quad (1)$$

$$[P]_i = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ mI^2 + K_{yy} & K_{yz} & 0 & 0 & 1 & 0 & 0 & 0 \\ K_{zy} & mI^2 + K_{zz} & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -I_p I^2 & I_t I^2 & 0 & 0 & 1 & 0 \\ 0 & 0 & I_t I^2 & -I_p I^2 & 0 & 0 & 0 & 1 \end{bmatrix}$$

where $\{\mathbf{S}\}_i$ is state vector, $[P]_i$ is the point matrix for i th mass vibrating in a normal mode with frequency I , m is mass and K is stiffness of bearing. I_p and I_t is pole inertia moment and transverse moment, respectively.

- Field matrix: The solution of the governing differential equations of an uniform beam segment connecting i th mass from differential equation of elastic curve and area moment method are written in a transfermatrix as equation (2) [3].

$$\{\mathbf{S}\}_{i+1}^L = [F]_i \{\mathbf{S}\}_i^R \quad (2)$$

$$[F]_i = \begin{bmatrix} 1 & 0 & 0 & L & L/GAk_s - L^3/6EI & 0 & E/2EI & 0 \\ 0 & 1 & 0 & 0 & 0 & L/GAk_s - L^3/6EI & 0 & E/2EI \\ 0 & 0 & 1 & 0 & 0 & L^2/2EI & 0 & L/EI \\ 0 & 0 & 0 & 1 & -L^2/EI & 0 & L/EI & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -L & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & L & 0 & 1 \end{bmatrix}$$

where $[F]_i$ is the field matrix for the i th massless elastic shaft and E is the Young' s modulus. I is inertia of moment and L is the length between the lumped mass.

- Overall transfer matrix: The 4 by 4 overall transfer matrix from station 1 to n th station in the over all system is constructed by the product of all field and point matrices in series. It can be written as equation (3).

$$\{\mathbf{S}\}_n^R = [P]_n [F]_{n-1} [P]_{n-1} \cdots [P]_1 [F]_1 [P]_1 \{\mathbf{S}\}_1^L \quad (3)$$

An algebraic equation is obtained by imposing the free boundary condition in equation (3); $V=0$, $M=0$ at both lumped mass 1 and n th. The solution of the eigenvalue equation can be obtained by a root-finding technique for the value of I . Once the natural frequency is founded, the corresponding mode shape can be determined by unit slope $w_0=1$ at the first mass.

B. Structural finite element analysis

The vibration equation for a mechanical undamped system based on the principle of Hamilton can be expressed in matrix notation when the free vibration is periodic [5]:

$$([K] - \lambda^2 [M])\{\phi\} = \{0\} \quad (4)$$

where eigenvector $\{\phi\}$ presents the mode shape at the natural angular velocity, $[M]$ and $[K]$ are the global mass matrix and stiffness matrix, respectively. The above equation has a no-zero solution. From the roots of the determinant in the parenthesis of equation (4), the natural frequencies and modes of rotor can be determined.

IV. COMPUTATION AND MEASUREMENT RESULTS

In TMM, the model is divided into 28 lumped masses and then they are constructed by connecting the each mass as an uniform beam segment. For FEM using ANSYS, about 1992 hexahedron-shaped elements are used for 3-D modeling of solid structure that is defined by eight nodes having three DOF at each node. The computation for solving the natural frequencies is implemented with the aid of a Pentium Celeron433 computer.

Fig. 2 shows the measured frequency response functions sum obtained from the experimental modal analysis. The inertance in the y-axis is the transfer function resulting from the relationship between impulsive excitation and acceleration signal. The natural frequencies exist within the audible sound range.

Table II shows the comparison of calculation and

measurement for the natural frequencies of the rotor. The analysis results obtained by FEM and TMM are in good agreement with the experimental values. The computational error is mainly attributed to the simplicity of the computing model. The 4th natural frequency is greater than the other mode and it is due to the lamination along the stack length.

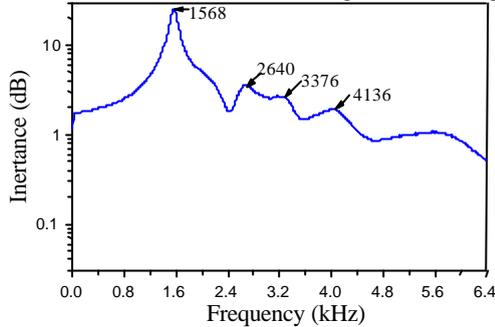


Fig. 2. Measured frequency response functions

TABLE II

NATURAL FREQUENCY COMPARISON

Mode	TMM(Hz)		FEM(Hz)		Measured frequency
	Frequency	Error	Frequency	Error	
1st	1588	1.28	1578	0.64	1568
2nd	2812	6.52	2535	3.98	2640
3rd	3501	3.70	3327	1.45	3376
4th	4532	9.57	4686	13.30	4136

TABLE III

COMPUTATION GAIN AND NODE NUMBER COMPARISON

Item	Number of node	Computing time (sec)	Computation gain(FEM/TMM)	Average Error for experiment
TM	28	40	1	5.02
M				
FEM	2606	263	6.6	4.84

Table III shows the computation gain and the computing time in each analysis method. The computation gain is defined as the proportion of computing time using FEM to using TMM. The method of TMM can reduce the computing time about six times over FEM.

Fig. 3 shows the mode shapes for the rotor with corresponding to the natural frequencies in 2D plane. The displacement value of mode shape displayed in Fig. 3 is normalized. The calculations are based on a 3-D TMM structural analysis considering the bearing of rotor. The 2nd and 3rd modes have one and two nodal points, respectively. If rotor bearings are placed in these nodal points, the vibration tends to be increased. Therefore, the bearing position should avoid these nodal points.

Each rotor behavior that corresponds to a resonant frequency is displayed on 3D spatial plane is shown in Fig. 4. These mode shapes present the locus of rotor center along stack length when the rotor is rotating. There are two whirling modes of a rigid rotor in flexible end bearings. Fig. 4(a) is the translatory whirling motion and Fig. 4(b) is the conical whirling motion of rotor. The first and second modes do not produce deformation of rotor along the shaft elastic axis,

because the behavior of rigid body rotor is caused by the stiffness of the rotor bearing supporting both end of rotor. These two modes become the origin that produces the eccentricity of rotor in an electric machine. In order to reduce vibration in these modes, it is important to select the bearing with appropriate stiffness based on the modal analysis.

An accurate analysis of resonant frequencies and the rotor design can be achieved from proposed analysis method. In addition, the analysis about modal analysis will allow us to propose a less noisy structure and resonant speed.

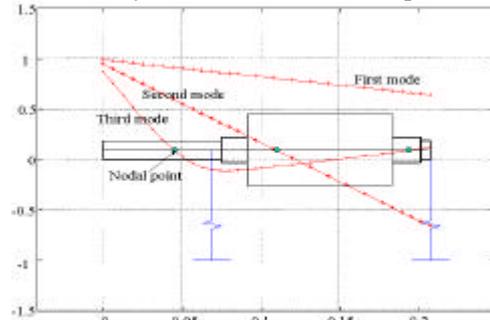
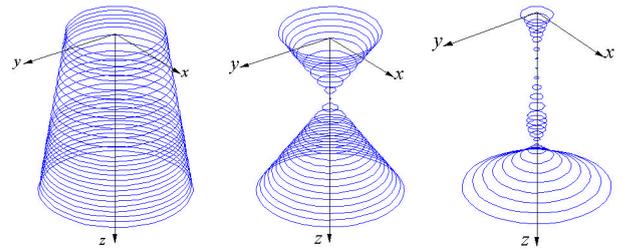


Fig. 3. Mode shapes of the SRM rotor in 2-D



(a) Mode at 1588 Hz (a) Mode at 2811 Hz (a) Mode at 3501 Hz

Fig. 4. 3-D whirling mode with corresponding to natural frequency of rotor

V. CONCLUSION

This work presents the natural frequencies and their mode shapes of the rotor in SRM. Transfer matrix method is used to reduce the calculation time. For the calculation time and accurate analysis, the validation of the proposed method is verified by the comparison of the experiment and FEM. In addition, the effect of the three dimensional mode shapes of rotor on the vibration characteristic is investigated and described. Due to the speed of the computation, the method proposed in this paper is more practical for estimation of the natural frequencies of the rotor and regulating the designing scheme when designing the rotor in SRM.

VI. ACKNOWLEDGMENT

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VII. REFERENCES

- [1] C. Picod, M. Besbes, Camus and M. Gabsi, "Influence of stator geometry upon vibratory behaviour and electromagnetic

- performance of switched reluctance motors," *IEE Proc. Electr Power Appl.* Vol. 145, pp. 462-468. 1998.
- [2] R. S. Colby, F. M. Mottier and T. J. E. Miller, "Vibration modes and acoustic noise in a four-phase switched reluctance motor," *IEEE Trans. on Industry Application*, vol. 32, pp. 1357-1364, 1996.
- [3] J. S. Rao, *Rotor Dynamics*, John Wiley & Sons, 1991.
- [4] C. G. C. Neves *et al.*, "Vibration behavior of switched reluctance motors by simulation and experimental procedures," *IEEE Trans. Magn.* vol. 34, pp. 3158-3136, 1998.
- [5] P. Pillay and W. Cai, "An investigation into vibration in switched reluctance motors," *IEEE Trans. Industry Appl.*, vol. 35, pp. 589-596. 1999.
- [6] C. Yongxiao, W. Jianhua and H Jun, "Analysis calculation of natural frequencies of stator of switched reluctance motor," *IEE EMD97*, pp. 81-84. 1997