

# Design and Analysis of Air Core type Permanent Magnet Linear Brushless Motor by using Equivalent Magnetizing Current

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**Abstract**- This paper presents design and steady state analysis method of air core type Permanent Magnet (PM) linear brushless motor based on the Equivalent Magnetizing Current (EMC) method. The phenomena of air core type motor by the variety of coil shape under the constant Magneto Motive Force (MMF) has been analyzed and the analysis process is applied to the design and static state analysis considering commutation thrust ripple. The validity of the proposed technique is confirmed with 2-D Finite Element (FE) analysis and experimental results.

**Index Terms**--EMC, air core type permanent magnet linear motor, commutation thrust ripple, 2-D FE.

## I. INTRODUCTION

Permanent Magnet (PM) linear motors are used extensively in speed and position controlled drive system. For example, in the factory automation and equipment of a semiconductor manufacture, for which linear brushless PM motor drives could offer significant advantages, in terms of efficiency, speed control and positional accuracy. However, in slotted PM linear motor, there are cogging force, due to slotting and finite length of the moving parts, which have different wavelengths [1-2]. In order to minimize cogging force, the additional process, such as skewing and optimally disposing the magnets and the optimizing the length of the armature coil, is required [2-3].

For this reason, in order to improve the accuracy in the speed and position control of linear brushless PM motor, an air-core type PM linear brushless motor that is without cogging force is strongly required. To obtain a precise design result of air core PM linear brushless motor it is necessary to analyze magnetic field in airgap and coil area. Recently, many numerical methods have been proposed to analyze machine performance through field analysis. As one of the numerical methods, the Finite Element (FE) method allows an accurate analysis of electrical machines and can consider geometric details and the non-linearity of magnetic material [2][4]. However, it is time consuming and unsuitable for the initial design stages and optimization from dynamic analysis. Therefore, an Equivalent Magnetizing Current (EMC) method, which is to solve the Poisson equation by replacing independent sources with the distribution of EMC, is usually used for magnetic field analysis because of their fast and flexible computation [5-6].

In linear PM machines, skewing of either the magnet or armature winding is usually incorporated in order to reduce the thrust ripple as well as noise and vibration [7-8]. The skewing effect is analyzed by 2-D FE analysis or analytical

method considering lumped parameters that involve formulation of expressions for airgap permeances, Magneto Motive Force (MMF) and flux density distribution. The precise analysis of skewing effect using analytical method taken into account lumped parameters is difficult because the analysis of flux distribution in airgap and coil region is different.

This paper presents a novel design process to improve design quality and a static state analysis that is phase commutation thrust ripple in static state taken into account the variation of relative pole position by using EMC. In order to minimize commutation thrust ripple, magnet skewing and variation of pole ratio are applied. In the analysis of skewing effects, a novel method that substitutes the spatial distribution of EMC for skewing magnet is adopted. The proposed analysis and design process is verified by comparison with the 2-D FE analysis and experimental results.

## II. MAGNETIC FIELD ANALYSIS BY USING EMC

### A. Motor topologies

An air core type PM linear brushless motor consists of primary and secondary which is moving part. The linear motors are usually classified into Moving Magnet (MM) type and Moving Coil (MC) type [2][5]. Fig.1 shows the topologies of MC and air core type linear motor studied in this paper. It consists of a double side the PM and an air core type moving armature which is concentrated winding having three isolated phase sets. The cross section of air core type linear motor is shown in Fig. 1b. Three phase coils are displaced symmetrically at  $120^\circ$  from each other and the current waveform in the phases is a  $120^\circ$  squarewave. Exactly two phases are conducting at any and every instant [6-7].

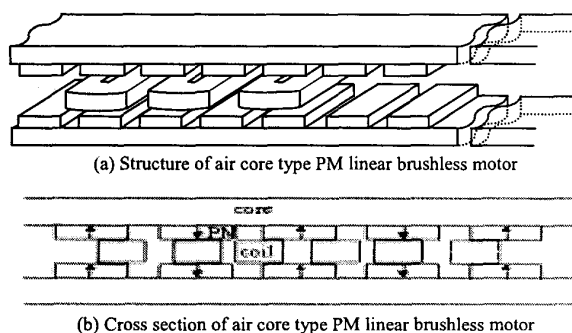


Fig. 1. Motor topology.

### B. Magnetic field analysis

Fig. 2 shows an analysis model for the magnetic field of air core type linear PM motor using EMC method.

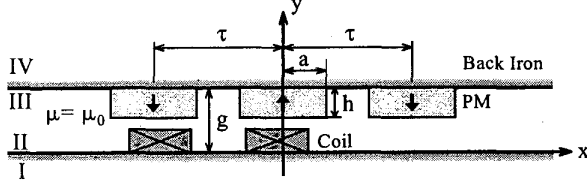


Fig. 2. Analysis model using EMC method.

Since the double side PM linear motor has symmetric structure along y-axis, only one side is selected as analysis region and as follows assumed to simplify the 2-D analysis and compute the magnetic fields of each region [5-6].

- All regions are extended infinitely in the  $\pm x$  direction and PM's are magnetized in the  $\pm y$  direction.
- Permanent magnets are periodically distributed along x-axis.
- Permeability of back iron is infinite.
- There is no variation along z-axis.

The final whole magnetic field can be obtained by the superposition of magnetic fields produced by PM and armature current.

#### 1) Magnetic field by armature current

For unexcited PM's, the analysis model is divided into four regions as shown in Fig. 3 and the governing equations of each regions derived from Maxwell's equations are as follows [1][4].

$$\frac{\partial^2 A(x, y)}{\partial x^2} + \frac{\partial^2 A(x, y)}{\partial y^2} = 0 \quad \text{Region I, III, IV} \quad (1)$$

$$\frac{\partial^2 A(x, y)}{\partial x^2} + \frac{\partial^2 A(x, y)}{\partial y^2} = -\mu_0 J(x) \quad \text{Region II} \quad (2)$$

where  $A$  is the z-axis component of magnetic vector potential and  $J$  is armature current density.

The magnetic fields induced by armature winding coils can be calculated by applying boundary conditions to the tangential components of magnetic field intensity and normal components of magnetic flux density at each boundaries in appropriate regions. These are as follows [5].

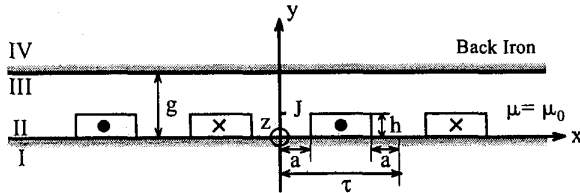


Fig. 3. Arbitrary current model.

$$B_{II}^x = -\mu_0 \sum_{n=1,3}^{\infty} \left( \frac{\sinh(nk(g-h))}{\sinh(nkg)} \right) \sinh(nky) \cdot \frac{b_n}{nk} \cdot \sin(nkx) \quad (3)$$

$$B_{II}^y = -\mu_0 \sum_{n=1,3}^{\infty} \left( 1 - \frac{\sinh(nk(g-h))}{\sinh(nkg)} \cosh(nky) \right) \cdot \frac{b_n}{nk} \cdot \cos(nkx) \quad (4)$$

$$B_{III}^x = \frac{\mu_0}{2} \sum_{n=1,3}^{\infty} \left( \frac{\sinh(nkh)}{\sinh(nkg)} \right) \left( \frac{e^{nky}}{e^{nkg}} - \frac{e^{nkg}}{e^{nky}} \right) \cdot \frac{b_n}{nk} \cdot \sin(nkx) \quad (5)$$

$$B_{III}^y = -\frac{\mu_0}{2} \sum_{n=1,3}^{\infty} \left( \frac{\sinh(nkh)}{\sinh(nkg)} \right) \left( \frac{e^{nky}}{e^{nkg}} + \frac{e^{nkg}}{e^{nky}} \right) \cdot \frac{b_n}{nk} \cdot \cos(nkx) \quad (6)$$

where  $b_n$  is  $\frac{4J_0}{n\pi} \cos(nka)$  and  $k$  is  $\frac{\pi}{\tau}$ .

#### 2) Magnetic field by PM

Assuming that the armature windings are not excited, the analysis model can be considered as Fig. 4a. In Fig. 4b, PM's are replaced by the EMC distribution and the characteristic equations of each region are as follows [5].

$$\frac{\partial^2 A(x, y)}{\partial x^2} + \frac{\partial^2 A(x, y)}{\partial y^2} = -\mu_0 J_p(x) \quad \text{Region III} \quad (7)$$

The EMC distribution by PM's,  $J_p(x)$ , for the region III, is described as the Fourier series [5].

$$J_p(x) = \sum_{n=1,3}^{\infty} b_{pn} \cdot \sin(nkx) \quad (8)$$

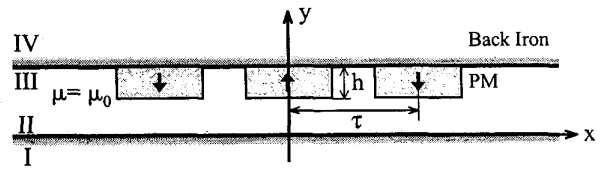
where  $b_{pn} = \frac{4J_{pm}}{n\pi} [\cos(nka) - \cos nk(a + \delta)]$ ,  $\delta$  indicates an arbitrary value which approaches zero and the current density of PM,  $J_{pm}$ , can be expressed by the magnetization  $M$ .

By applying the boundary condition to the interfaces between different material regions, the characteristic equations given in (7) can be solved.

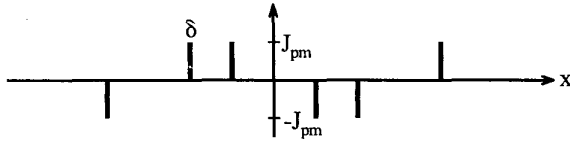
$$B_{II}^x = \frac{\mu_0}{2} \sum_{n=1,3}^{\infty} \left( \frac{\sinh(nkh)}{\sinh(nkg)} \right) \left( \frac{e^{nky}}{e^{nkg}} - \frac{e^{nkg}}{e^{nky}} \right) \cdot \frac{b_n}{nk} \cdot \sin(nkx) \quad (9)$$

$$B_{II}^y = -\frac{\mu_0}{2} \sum_{n=1,3}^{\infty} \left( \frac{\sinh(nkh)}{\sinh(nkg)} \right) \left( \frac{e^{nky}}{e^{nkg}} + \frac{e^{nkg}}{e^{nky}} \right) \cdot \frac{b_n}{nk} \cdot \cos(nkx) \quad (10)$$

The resultant magnetic field is obtained by superposition of the magnetic field caused by PM and armature current.



(a) Magnets distribution



(b) Equivalent Magnetizing Current (EMC) of magnets

Fig. 4. Magnetic field analysis model by only magnet.

### III. DESIGN AND CHARACTERISTIC ANALYSIS

The magnet specifications such as coil area width, thickness and MMF are important parameters to determine motor efficiency and power density per weight for all PM. In the case of air core type PM motors, although a magnet specification and MMF is fixed, thrust is varied with shape of coil area. There is an optimal coil shape under the same MMF for generating the maximum thrust. The thrust is computed by Lorentz force law and magnetic flux density in airgap is calculated with magnetic vector potential by EMC method. In computing of thrust and Electro-Motive Force (EMF), the armature winding area is divided into very small regions and the flux linkage of the each element is computed by the EMC method. The EMC by PM is moving then EMF is calculated from flux linkage and thrust is calculated by armature current with flux density in accordance with pole position. The calculation process is as follows.

$$F = \sum_{i=1}^n F_i^{(e)} = \sum_{i=1}^n J_i^{(e)} \cdot S_i^{(e)} \cdot B_y^{(e)} \cdot L \quad (11)$$

$$E = \omega_e N \sum_{i=1}^n \phi_i^{(e)} \cdot \sin p\theta \quad (12)$$

where  $S_i^{(e)}$  is the subdivided element of armature winding region,  $L$  is z-axis length of machine,  $\omega_e$  is electric angular frequency,  $N$  is turns per phase,  $p$  is pole pair and  $\theta$  is relative electric angle of PM and armature winding.

Fig. 5 shows the design process of air core type PM linear brushless motor by using EMC method. The shape variation of armature winding coil area for computing the maximum thrust under fixed design parameters process is shown in Fig. 6. The armature winding area is divided into very small

regions, and the flux density of the each subdivision region is computed by the EMC. If the cross sectional shape of the coil area changes, the magnitude and distribution of magnetic flux density is varied in the magnetic airgap, the generated thrust would be also varied in spite of the constant MMF.

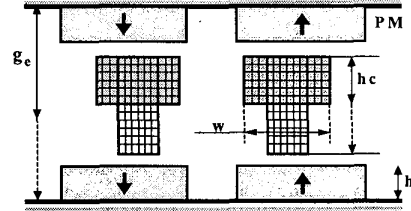


Fig. 6. The shape variation of winding coil area.

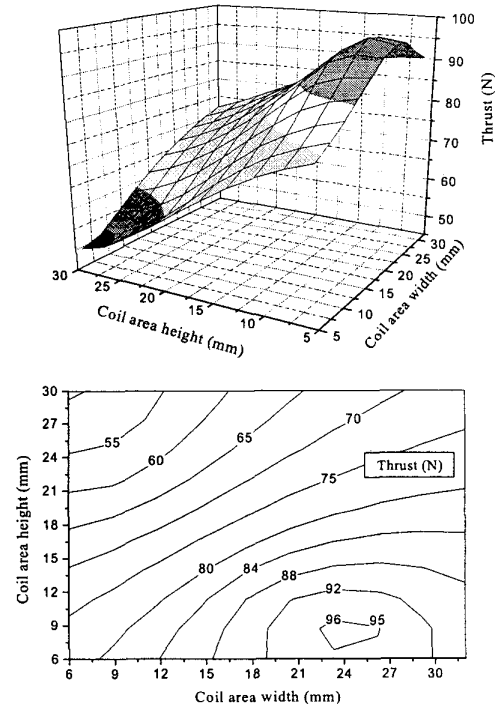


Fig. 7. Thrust characteristic according to shape variation.

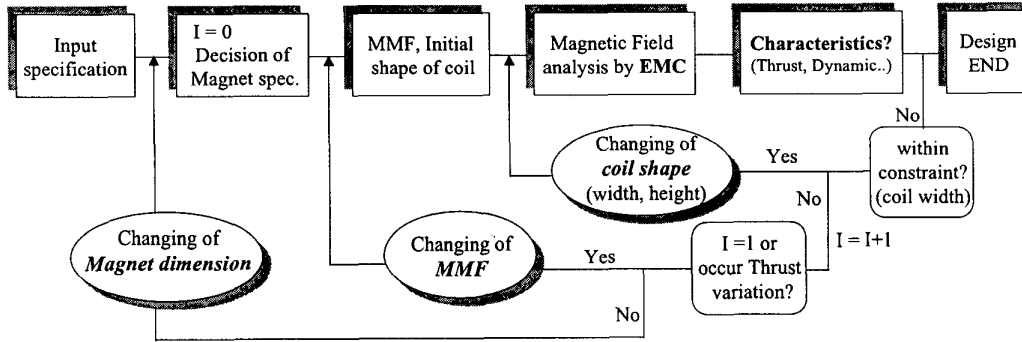


Fig. 5. Design process of air core type PM linear brushless motor by EMC.

TABLE I  
SPECIFICATION OF DESIGN RESULT

Line voltage	100 (V)	Current	1.8 (A)
Stack length	100 (mm)	Thrust	100 (N)
Back iron thickness	10 (mm)	Coil	
<b>Magnet</b>		Core area	214 (mm <sup>2</sup> )
Residual flux density	1.15 (T)	Turn number	302
Width	35 (mm)	Coil area width	23 (mm)
Thickness	5 (mm)	Coil area height	9.3 (mm)

Fig. 7 shows the thrust characteristic by the variation of winding area width and height under constant MMF and armature winding area. As the width of winding area is increased, the magnetic airgap is reduced and thrust is increased. However, thrust is found to be reduced for over the specific length. The specification of air core type PM linear brushless motor from the design process is shown in TABLE I.

#### IV. COMPARISON OF 2-D FE ANALYSIS AND EXPERIMENTAL RESULTS

The appropriateness of the presented analysis and design method of air core type PM linear brushless motor, which is using EMC, is verified through comparison 2-D FE analysis with experimental data.

##### A. Comparison with 2-D FE Analysis

The FE analysis is known as an accurate analysis method for electrical machines and allows consideration on geometric details and the non-linearity of magnetic material [2][4]. Therefore, the proposed EMC method is verified with FE analysis result.

The Equi-potential distribution of the linear brushless PM motor is shown in Fig. 8. In the case of air core type PM linear brushless motor, the effective airgap length is increased since coil winding area is considered as magnetic airgap. Therefore, magnetic flux density in each position of winding area is different accordingly. Fig. 9 shows the flux density distribution in mechanical airgap and the center of magnetic airgap which is center of winding coil area. The flux density in the center of mechanical airgap has a rectangular characteristic. However, the flux density of magnetic airgap center has a sinusoidal waveform due to increased magnetic airgap length. It is fringing effect to occur in magnet edge and shown that the characteristics of flux density by the two methods, EMC method and FE analysis, is in good agreement.

The characteristic of velocity EMF by moving PM is shown in Fig. 10. The shape of EMF distribution in each phase is sinusoidal. It is due to flux distribution in magnetic airgap and raises a problem of commutation thrust ripple according to driving method in brushless motor.

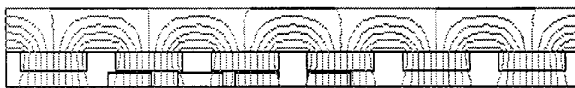


Fig. 8. Equi-potential distribution.

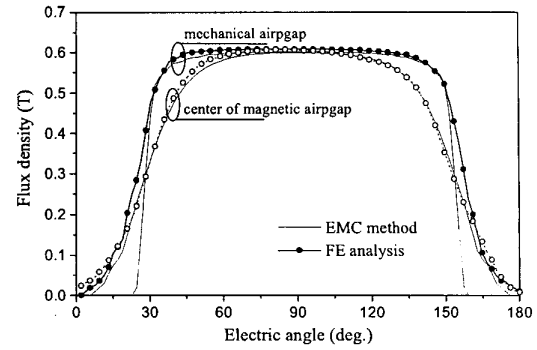


Fig. 9. Flux density distribution.

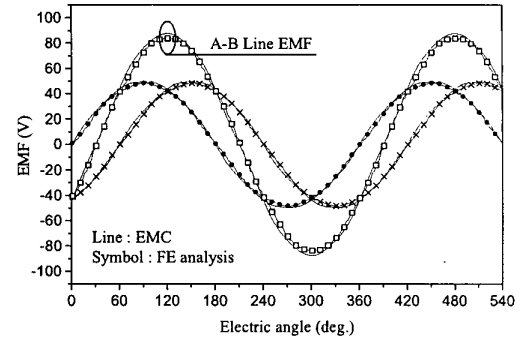


Fig. 10. Back EMF distribution.

The thrust calculation process by using EMC is shown in Fig. 11. It is shown that EMC by PM is moving and thrust can be calculated from flux linkage and armature current. In the case of FE analysis, moving line is applied technique then on time of armature current in each phase is a 120° and phase current is commutated by every 60° in electrical angle. Exactly two phases are conducting at any and every instant [7].

The result of thrust calculation by using EMC method and FE analysis are shown in Fig. 12. The phase current is considered square waveform, the other side characteristics of flux distribution has not perfect to square form. Therefore, thrust ripple during every 60° is generated by the characteristic of flux density distribution and the resultant thrust by FE analysis is somewhat less than that of EMC method. In FE analysis, the nonlinear characteristic of permeability in iron core is considered [4].

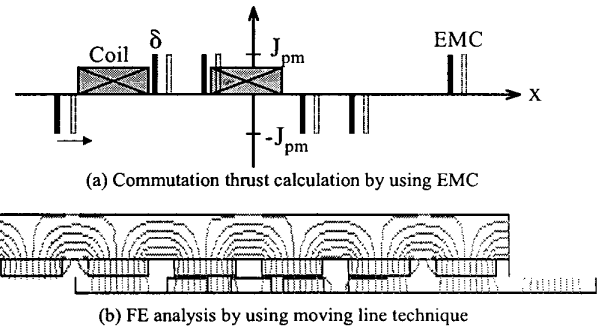


Fig. 11. Thrust calculation process during the on time.

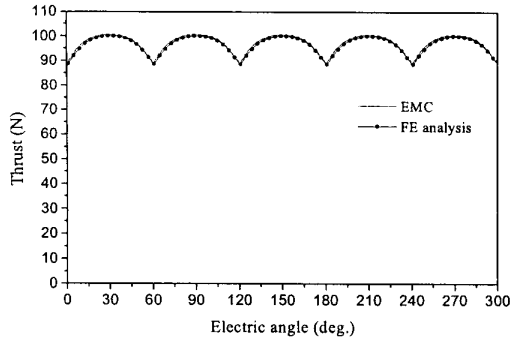


Fig. 12. Thrust distribution according to commutation period. ( $I=1.8$  A)

### B. Comparison with Experiment

The proposed EMC method is compared with experimental results. The system structure of air core type PM linear brushless motor is shown in Fig. 13.

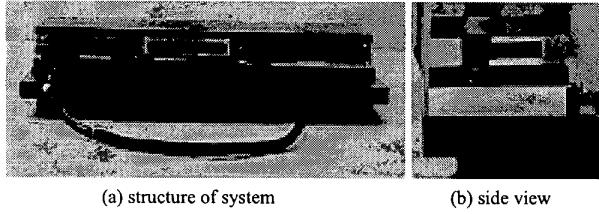


Fig. 13. The system of testing machine.

Fig. 14 shows the flux density distribution of the test machine for different airgap position and Fig. 15 shows velocity EMF. It is shown that an error of flux density magnitude is within about 3 percent so the proposed analysis. EMC method, is found in well agreement with experimental result. The estimated result of velocity EMF is shown in TABLE II. The EMF constant is good agreement with thrust constant by EMC method.

In Fig. 16, the characteristic of thrust according to pole position is compared with experimental for each analysis results. In the comparison, the measured thrust value is less than that of two analysis results. The reason is that experimental equipment consists of back iron core which is not lamination so it seems that leakage of flux is increased. The thrust ripple during  $60^\circ$  electric angle occurs and it is the same phenomena with analysis result.

From the comparison with the experimental results, the proposed design and analysis process by using EMC method is found to be verified for air core type PM linear brushless motor.

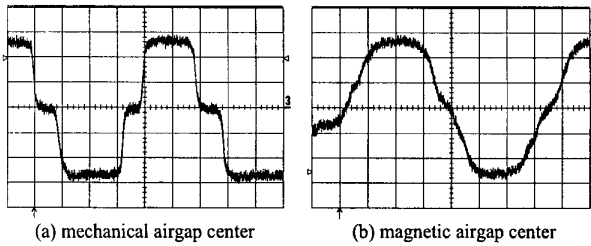


Fig. 14. Flux density distribution by Experiment. (0.2 T / div.)

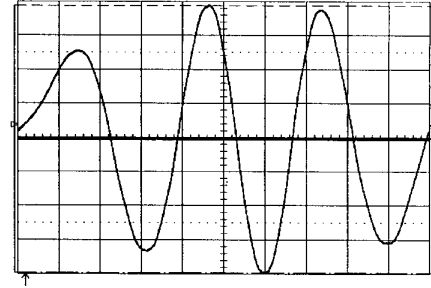


Fig. 15. Velocity EMF. (20 V / div.)

TABLE II  
THE RESULT OF VELOCITY EMF

EMF (V) (pk-pk)	Distance in a moving (mm)	Moving time (sec)	Velocity (m/sec)	EMF constant (Vsec/m)
17.34	100	0.6523	0.1533	56.56
38.12		0.294	0.3401	56.04
38.3		0.288	0.3472	55.16

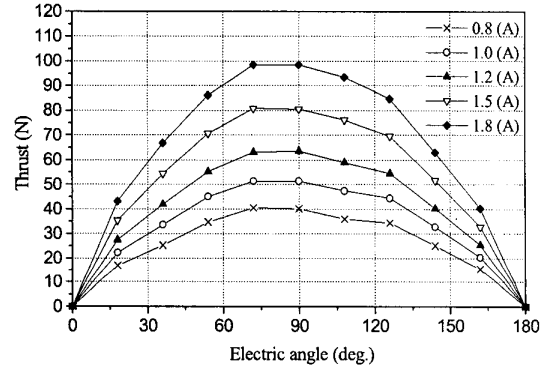


Fig. 16. Thrust distribution according to electrical angle.

### V. REDUCTION OF COMMUTATION THRUST RIPPLE

The variation of instantaneous thrust, which is summation of electromagnetic and cogging force, is generated with moving position and it decides both the average and the ripple component [8]. In the case of air core type PM linear motor, there is no cogging force because of slotless structure.

The presented model in this paper tends to have large electromagnetic force ripple that is commutation thrust ripple. Therefore, magnet skewing and variation of pole ratio are applied in order to minimize electromagnetic force ripple. In analysis method of skewing effects, a novel method that substitutes the spatial distribution of EMC and EMC segment for skewing magnet is adopted. Fig. 17 shows the spatial distribution of skewed magnet and analysis process by the spatial distribution of skewed segment EMC is shown in Fig. 18 where  $n$  is segment number of the skewed magnet.

The results of skew effect by using segment EMC method is shown in Fig. 19 and 20. If the phase current is considered square waveform entirely, thrust distribution depends on only the back EMF distribution. In the case of driven to squarewave current, skew effect for minimization of thrust

ripple is scarcely but the thrust ripple can be greatly reduced in the case of sinusoidal current. The analysis result of skew effect is shown in Fig. 19. Fig. 20 shows the thrust ripple during each phase on time with changing skew angle of PM.

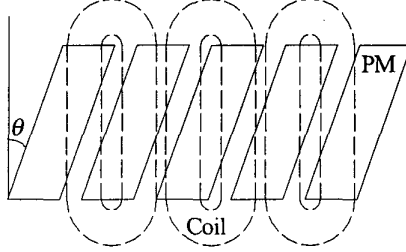


Fig. 17. A developed view of the skewed magnet.

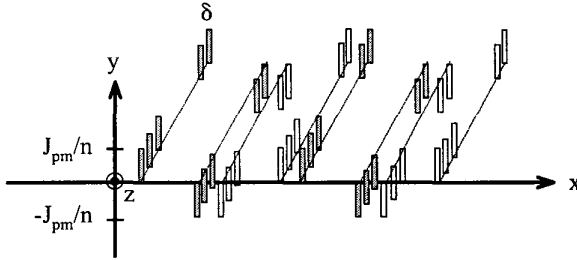


Fig. 18. The spatial distribution of skewed segment EMC.

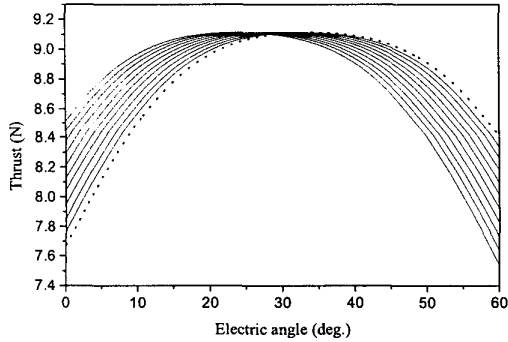


Fig. 19. Thrust distribution by using segment EMC.

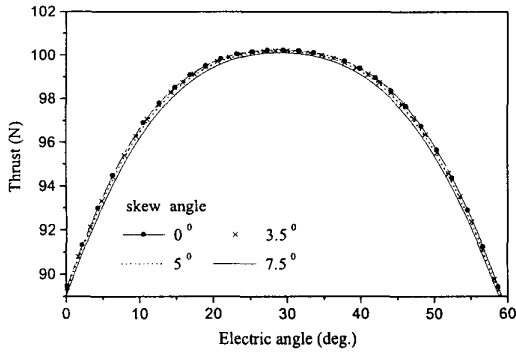


Fig. 20. Thrust ripple according to changing of skew angle.

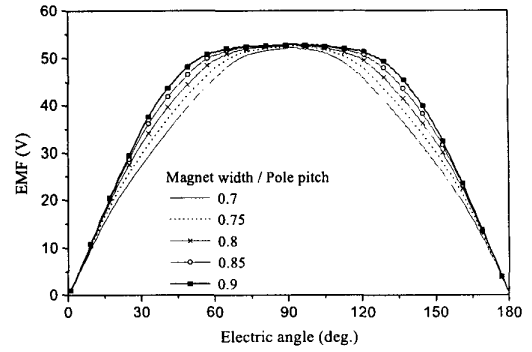


Fig. 21. EMF distribution according to pole ratio

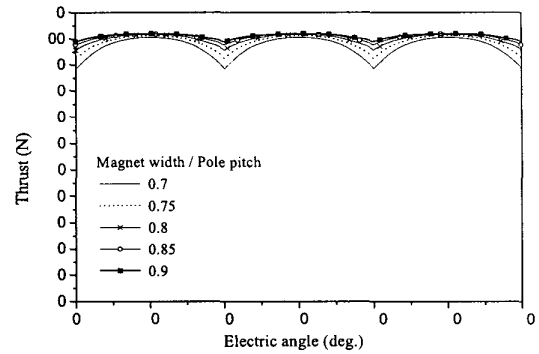


Fig. 22. Thrust distribution according to pole ratio

Fig. 21 shows the variation of back EMF distribution with varying pole ratio (magnet width / pole pitch). The shape of back EMF is approximated to trapezoidal when pole ratio is increased. Fig. 22 shows thrust ripple for different pole ratio. It should be noticed that the thrust ripple is slightly influenced by skewed magnet whereas variation of pole ratio is severely affected. This is mainly due to combination of current waveform and back EMF distribution.

## VI. CONCLUSION

Magnetic field distribution has a very significant effect on the characteristic analysis and design process, for the air core type PM linear brushless motor. An EMC method has been developed to aid the magnetic field analysis of air core type PM linear brushless motors. It is based on the calculation of the spatial distribution magnetic field when the PM's are replaced by the EMC distribution.

In this paper, a design process and characteristics analysis by using EMC method are presented and the results are verified by the 2-D FE analysis and experimental results. The shape of winding coil area has a significant effect on the thrust. The effect of various coil shapes under constant MMF has been investigated and thrust ripple taken into account a driving manner is analyzed by EMC method that is the moving of magnetization.

By comparing the experimental results, the reasonable agreement of proposed process has been obtained by EMC

method. The EMC method can gain reasonable analysis results to be an appropriate analysis tool.

#### ACKNOWLEDGMENT

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