

# Force Distribution Analysis in Link of Interior Permanent Magnet Synchronous Motor using FEM

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## Abstract

In this paper, the radial force acting on surface of the link of Interior Permanent Magnet type synchronous motor(IPM) was calculated by Equivalent Magnetizing Current(EMC) and the harmonics analyzed by Discrete Fourier Transform(DFT). Additionally mathematical modelling of the link for the mechanical characteristic is modelled as supported beam at both end. Mode shape and natural frequency is determined.

A IPM has a advantage of high power per volume, but it has a weaken mechanical structure in the link. The aim of this study is to calculate reliable exciting force and characteristics of mechanical structure in the link of IPM.

## 1. Introduction

Comparing the brushless motor with the other motor has many merits, demand of a IPM steadily increase in all around industry which use control and power machines. Developing new material of permanent magnet and technical improvement show the tendency to accomplish high performance and light goods. Accordingly, increase of high energy density gives rise to a noise and vibration, so the performance of a motor is deteriorated.

The source of a noise and vibration can be classified electromagnetic and mechanical origin. In case of former, electromagnetic force in electric machines occurs torque of a motor, and exciting force become radial force acting on stator which produces a vibration. Specially, if the frequency of exciting force is in the near of frequency band of natural frequency of a structure, a mechanical structure get highly vibration due to resonance. Hence it reduce lifetime of a motor and is transferred to objects in close system. The noise is and audible sound with has an unpleasant effect on human beings.

The methods for the force calculation can be classified into several methods. It is the Maxwell stress, virtual work principle, EMC and magnetic moment method. In generally, Maxwell stress method by Finite Element Method(FEM) has different results of force according to the integral path. If the

integral path of Maxwell stress is chosen very closely to the iron-air surface, the error on force by elements is refined. However it is a tedious task and one do not has reliability.[1]

The EMC method that magnetizing component of magnetic core is replaced by EMC. On the other hand, the link of a IPM can contribute to designed, the link must be considered to be due to a shortcoming of mechanical weakness.

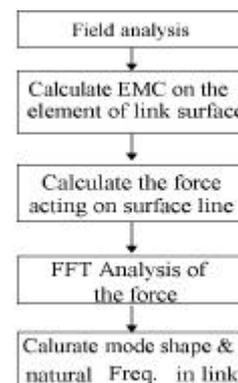
In this paper, the EMC method used for evaluating variation of force calculation acting on the surface of link according to teeth width. The model of link is presented by simply beam, and mode shape, natural frequency is studied.

## 2. Analysis of Exciting force

### 2.1 Calculation procedure

The magnetic field is obtained by calculating FEM, and force distribution acting on surface is calculated by EMC. Next, it is compared harmonic of exciting force by using DFT with natural frequency. The process of analysis by using EMC are shown in Table 1.

Table 1. analysis flow



### 2.2 Field analysis

Magnetostatic field governing equation by field variable with vector potential is equal to equation (1).[2]

$$\frac{1}{\mu} \nabla^2 \vec{A} = \vec{J} + \vec{J}_m \quad (1)$$

$$\vec{J}_m = \frac{1}{\mu_0} \nabla \times \vec{M}$$

Where  $\vec{A}$  is Magnetic vector potential,  $\vec{M}$  is the magnetization,  $\mu_0$  is the magnetic permeability of vacuum,  $\mu$  is the magnetic permeability of core.

Fig. 1.(a) is cross section of model to analysis of IPM by using FEM. Fig. 1.(b) show structure of the link.

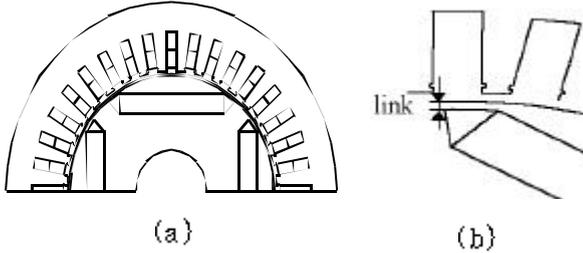


Fig. 1 cross section of IPM and link

### 2.3 Equivalent magnetizing current

Each elements, shown in Fig2 have different material. Magnetizing current exists on element border-line.

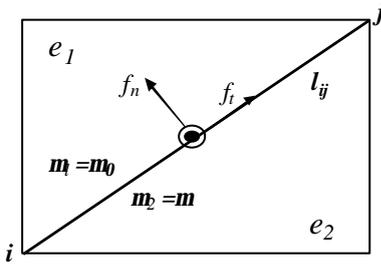


Fig. 2 magnetizing current between two material

The interior magnetizing current in core is cancelled so magnetizing current only distribute on the element surface of different material.

The current  $I_m$  on the line forming element  $e_1$  and  $e_2$  in Fig2 can be expressed as follows equation (2).

$$\begin{aligned} I_m &= \frac{1}{\mu_0} \int_s \nabla \times \vec{M} \cdot \vec{ds} \\ &= \frac{1}{\mu_0} (M_{1t} - M_{2t}) l_{ij} \end{aligned} \quad (2)$$

The equation (2) substitute the equation (3), for simplicity, we obtain current  $I_m$  on the line  $i, j$  with length  $l_{ij}$  become equation (4).

$$\vec{B} = \mu_0 \vec{H} + \vec{M} \quad (3)$$

$$I_m = \frac{1}{\mu_0} (B_{1t} - B_{2t}) l_{ij} \quad (4)$$

### 2.4 Radial force in the link

The force on the length  $l_{ij}$  with current distribution by Lorenz's law is follow equation

$$\vec{f}_{ij} = \vec{I}_m \times \vec{B}_{ext} \quad (5)$$

Where the sum of the external field due to source and the self field due to the magnetizing current in core is  $B_{ext}$ . Flux density value of  $B_{ext}$  is given as the average value for each element.[1],[3]

### 2.5 Analysis of frequency

The radial force acting on surface of the link is expanded by DFT that follow equation (6).

$$F(\omega t) = a_0 + \sum_{k=1}^{\frac{N}{2}-1} (a_k \cos k\omega t + b_k \sin k\omega t) \quad (6)$$

$$a_0 = \frac{2}{N} \sum_{i=0}^{\frac{N}{2}} NF(\omega t_i)$$

$$a_k = \frac{2}{N} \sum_{i=0}^{\frac{N}{2}} NF(\omega t_i) \cos k\omega t_i$$

$$b_k = \frac{2}{N} \sum_{i=0}^{\frac{N}{2}} NF(\omega t_i) \sin k\omega t_i$$

Where  $N$  is one periodic of the radial force.

## 3. Vibration analysis in Link

Assuming an object is very small mass element of infinite distribute all structure of an body. Each of infinite number of points of system have multidegree of freedom system. This distribute parameter system is analyzed by analytic method. The vibration analysis model is like following Fig. 3.(b) which is supported beam at both end.

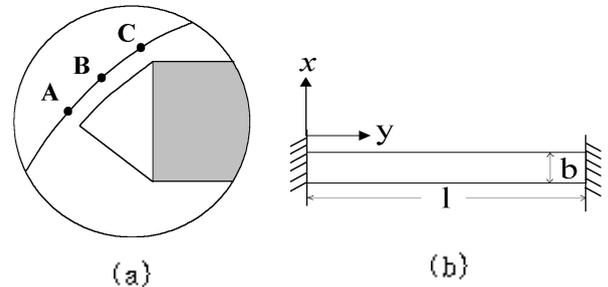


Fig. 3 vibration analysis model

It is assumed that rotor core is rigid body except the link, and link mass is very smaller than rotor mass for modelling the link.

Displacement  $w(x, t)$  of the link movement has

perpendicular at length direction. Exciting force applied uniformly to length. Disregarding shear deformation governing equation of beam is given by equation (7).[4]

$$\rho S(x) \frac{\partial^2 w(x, t)}{\partial t^2} + \frac{\partial^2}{\partial x^2} [EI(x) \frac{\partial^2 w(x, t)}{\partial x^2}] = f(x, t) \quad (7)$$

Where  $w$  is displacement in  $y$  axis,  $S(x)$  is area in link,  $\rho$  is density of core,  $E$  is Young's modulus,  $I(x)$  is the moment of inertia of the beam cross session about  $z$  axis.

To analysis of exact solution by using separation of variables, the assumption of analysis model are following

- A uniform and thin beam along length.
- Material of beam consist of linear, isotropy, homogeneous and elastic.
- Ignore effect of shear deformation and rotary inertia.

The free vibration solution can be found using the method of separation of variables, so initial condition for fixed both end is as follow.

$$w = 0, \quad \frac{\partial w}{\partial x} = 0$$

For free vibration  $f(x, t) = 0$  and  $EI(x)$ ,  $S(x)$  is constant, so the solution of motion from initial condition becomes.

$$X_n(x) = \left[ -\frac{\cosh \beta_n l - \cos \beta_n l}{\sinh \beta_n l - \sin \beta_n l} (\sinh \beta_n x - \sin \beta_n x) + \cosh \beta_n x - \cos \beta_n x \right] C_n$$

$$\beta = \frac{\omega^2}{c^2} = \frac{\rho A \omega^2}{EI} \quad (8)$$

Where  $\omega$  is natural frequency.

Table 2 shows physical coefficient to analysis of vibration.

Table 2 Physical coefficient

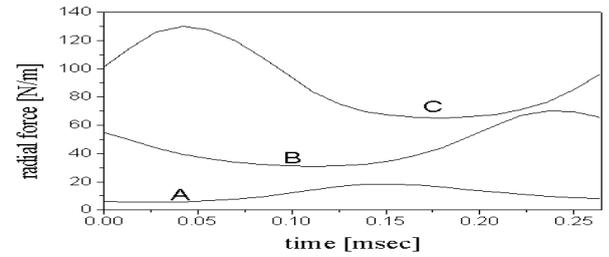
Young's modulus	205.8 [GPa]	length of the link	7.31 [mm]
core density	7650 [kg/m <sup>3</sup> ]	area of the link	0.5*1.0 [mm <sup>2</sup> ]
moment of inertia	3.316*10 <sup>-11</sup> [m <sup>4</sup> ]		

#### 4. Discussion of results

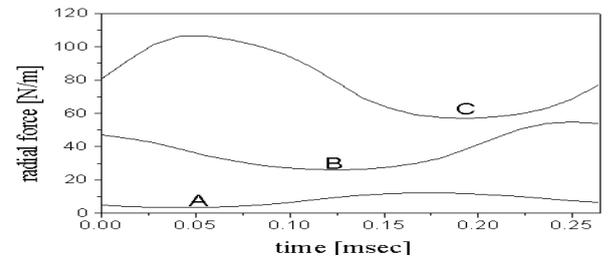
The local force of radial direction calculated at load

angle 90° which act on maximum attraction force in the link and armature current is 70[A] at the moment. The excited force on the surface of the link is calculated at three position in A, B, C as shown Fig 3(a). Slot pitch of analysis model dimension is 10.297[mm]

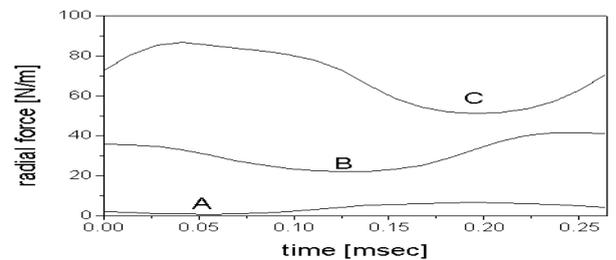
Fig. 4 shows variation of excited force according to change of slot width and also represents radial force distribution as time-dependent function. The reason why the radial force act on maximum at point C that the flux density of point C is maximum due to nonlinear in link, thus point C illustrated maximum stress point. The more slot width increase, the more radial force decrease, and increase half periodic time. Fig 5. represents the spectrum analysis by using DFT. The more slot width increase, the more wave deformation increase and include much harmonics. Among points of the link, the force of point A has many harmonics than other points.



(a) slot width: 5.5 [mm]

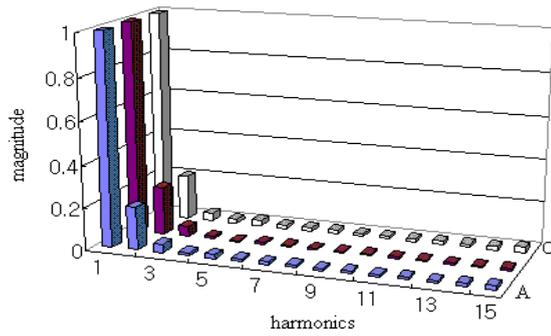


(b) slot width: 6.5 [mm]

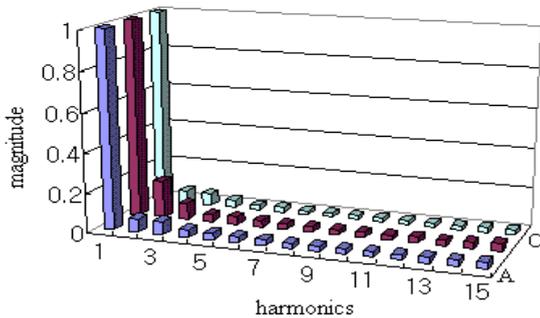


(c) slot width: 7.5 [mm]

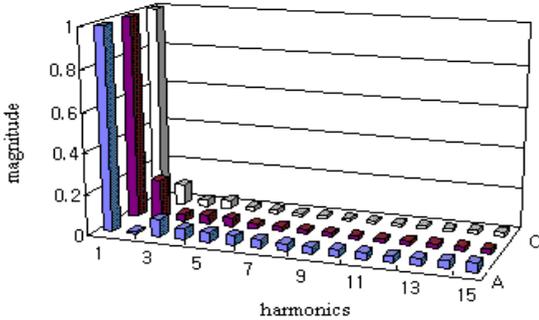
Fig. 4 radial force distribution



(a) slot width : 5.5 [mm]



(b) slot width : 6.5 [mm]



(c) slot width : 7.5 [mm]

Fig. 5 spectrum analysis of radial force

Fig 6. represents mode shape of the link from exact solution. Normalized mode shape is obtained by orthogonal characteristic of mode. Mathematical expression of the equation is following equation (9)

$$\int_0^l X_n(x) \cdot X_m(x) = 1 \quad (9)$$

Since the mass and damping ratio is small, the natural frequency of low order mode is high frequency as shown in Table 3.

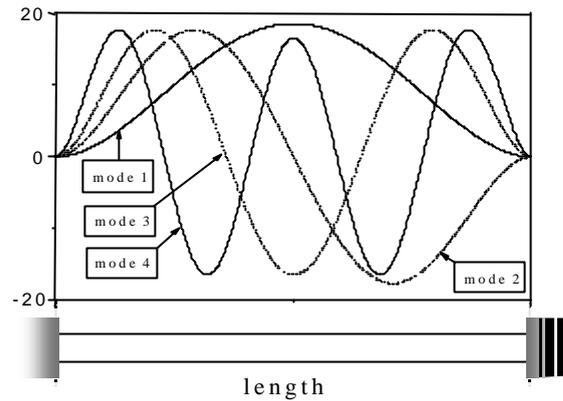


Fig. 6 analysis of mode shape

Table 3. natural frequency

mode	freq.	mode	freq.
1	2815 [kHz]	3	15214 [kHz]
2	7760 [kHz]	4	37569 [kHz]

When the rotor rotated at 7500[rpm] with 36 stator slot, the frequency of the radial force is 67.5[kHz] for the 15th harmonics. The result of this value is far from the natural frequency. And the safety operation is possible in mechanical aspect.

## 5. Conclusion

In this paper, it is evaluated by the method of EMC that exciting force acting on the link surface of a IPM. Effect of teeth width on harmonics of radial force distribution is explained. The mode shape and natural frequency in the link is calculated by utilizing beam model. and the results of the natural frequency are compared with harmonic of radial force.

## Acknowledgement

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## 6. References

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